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Modelling of Components of Building Structures Using Discrete Structures of Computer Science

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Abstract

Buildings are essential physical structures which serve several purposes including residential accommodation, factories, hospitals, laboratories, offices, schools, as well as religious places of worship such as church, mosque, shrine etc. A building is an aggregate of components such as roof, window, door and arch. From the perspective of computer graphics, each of these components can be formed from a point and a line. Several models dealing with aspects of building structures have been developed in the literature. Two of the popular generic models are the Tits Building model and the evolutionary computation (bioinformatics) model. While the former uses Discrete Structures of Computer Science (DSCS), specifically, set theory, graph theory and lattice theory, the latter is strictly based on genes derived from nucleotides in DNA Computing. In this paper, a novel DSCS model is presented using the qualitative equivalence behavior of a set of autonomous ordinary differential equations (ODEs). In the model, ODEs of the same kind are classified based on the phase portraits (i.e. geometrical shapes) generated from their equilibrium points. The model successfully presents a good representation of (aspects) of components of buildings, with focus on a door, and can be potentially simulated into computer software.

Keywords-Building structure, Discrete structure, Qualitative equivalence, Ordinary differential equations, phase portraits

I INTRODUCTION

Buildings are essential physical structures which serve several purposes including residential accommodation, factories, hospitals, laboratories, offices, schools, as well as religious place of worship such as church, mosque, shrine etc [1]. Historically, civil engineering encompasses buildings, roads and bridges, of which the design principles are somehow similar. However, in modern times, with increasing specialization, buildings are now normally classified under building technology/engineering [2] while civil engineering often refers to the design and construction of roads, bridges, rail tracks etc [3]. Structural engineering is a part of the larger civil engineering family which focuses on the study and design/calculation of appropriate loads, usually called bearing capacity, which a building/civil engineering structure will carry, and the balancing of these loads to enable the structure to be stable [4]. Normally, a structure cannot carry more than its bearing capacity. (Building) Structural analysis is invariably tied to engineering mechanics. This is of two kinds. While engineering statics deals with the study of a structure under dead or stationary load [5], engineering dynamics focuses on the analysis of a structure under applied or moving load [6]. In Nigeria, building/civil engineering has come of age. Examples of successful indigenous/Nigerian civil engineering firms over the years include T. A. Oni & Sons (the first indigenous construction company), L. A. O. Banjoko & Sons, Adebayo & Olatunbosun & Sons, and Dantata & Sawoe. Successful multinational firms that have operated on Nigerian soil include Julius Berger Plc, Cappa & Dalberto, China Civil Engineering Construction Company (CCECC), Arab Contractors, Solel Bonel, Taylor Woodrow, Strabag and Salini. A leading indigenous structural engineering firm is Ette & Aro Partners. Four key specialization areas of building technology are (i) Building Structures (which deals with stability of buildings) (ii) Building Maintenance, which focuses on keeping the building in its original state, as much as possible (iii) Building Services, which deals with building infrastructure such as lighting, sewage disposal system, plumbing works etc (iv) Construction Management, which deals with effective management strategies from inception of building to completion.

Generally, three major engineering processes are Design, Construction and (Performance) Testing/Evaluation [7, 8]. Design essentially involves preparation of the architectural structure; construction involves the translation of the architectural structure into real physical structure while performance testing deals with checking to ensure that functionalities of a building are functioning as desired, according to specification. A designer is a highly trained person who normally has skills to construct but a person who

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has the ability to construct may not necessarily be highly skilled in design. This phenomenon is the case with some typical indigenous construction companies, who could construct but not necessarily skilful in design. Generally, structural or design oversight may lead to disastrous consequences such as revealed via engineering forensics in the case of the collapse of the Kinzua Viaduct in 2003 [9]. This viaduct was initially built in North-Central Pennsylvania, USA, in 1882 but was rebuilt in 1900 before its eventual collapse over a century years later. A viaduct is a special bridge, especially a rail flyover, consisting of series of short spans.

The construction of a building is normally preceded by an architectural design, which in a sense, depicts the building model. This design invariably consists of geometrical structures composed of points, lines and derived shapes such as circles, spheres, rectangles etc. This aligns with computer graphics [10]. Furthermore, in the parlance of computer science, the geometric structures are data structures (DS), which may be fundamental (primitive) or derived (non-primitive) units of data that are manipulated in computer (software) design [11]. A set (real numbers, rational number, complex number etc) is a classic example of primitive DS while array, graph, stack, tree and queue are examples of non-primitive DS, built from the primitive structures. In the peculiar case in which a DS is binary in nature, or generates binary structures, it becomes a discrete structure. That is, a discrete structure is a data structure whereas a data structure is not necessarily a discrete structure. Examples of discrete structures are set, group, lattice and graph [12, 13, 14].

Mathematical) modeling refers to the act of translating physical problems into mathematical relationships. It plays a vital role in science, engineering and environmental sciences, in the design and analysis of physical systems which are intractable, costly or complex in nature [7, 15, 16]. In particular, models have historically been useful in studying the behaviour of environmental systems such as buildings, bridges and roads under applied loads [3, 4, 17]. This assists in determining the appropriate bearing capacity to be incorporated in the structural design. A classical example on the use of models to solve physical problems relates to the famous Konigsberg bridge problem which was solved by the great Swiss mathematician Leonhard Euler in the early 18th century. This pertains to the seven bridges in Konigsberg, Switzerland, which crossed the river Pregel. Euler proved mathematically, using a standard method of proof called 'proof by contradiction', that it is impossible to devise a route which crosses each bridge just once [12]. A field of mathematics which has been fundamentally useful in modeling is differential equations [18]. This field describes processes which change with time and so plays an important role in predicting or explaining the trend of changes in structural or dynamical systems.

Several discrete structure-related models have been developed in the literature. Two of the popular generic models are the Tits Building model [19] and the Bioinformatics model [20]. While the former makes use of set theory, lattice theory and graph theory, the former is based on evolutionary computation. The goal of the present paper is twofold. First, an abridged history of some of the major developments in the modelling and analysis of building and civil engineering structures is presented. Secondly, a novel model for theoretical construction of basic building components, with focus on a door, is presented based on discrete structure of computer science. Specifically, the qualitative equivalence behavior of a set of first order autonomous ordinary differential equations (ODEs), having a polynomial non-linear part, is considered [21]. This set-theoretic model is herein described as the qualitative equivalence model of buildings. In the model, the phase portraits (geometrical figures) resulting from the equilibrium points of the ODEs are used to model components of building structure such as wall, roof etc. In the parlance of discrete structures of computer science, these phase portraits are actually graphs consisting of nodes and edges. Simulation of models leads to the development of computer software. Examples of computer programs which have been used in performing the dynamical analysis of structures are SAP and NASTRAN. Popular computer graphics software include, among others the Microsoft Word, Corel draw, and LATEX.

In an earlier paper [22], it was shown that there is a marked similarity between the process of building a house (construction industry) and the process of building computer information system (computer industry). Specifically, it was shown that the systems development life cycle is applicable to both. In [23], some novel computer, and information and communication technology (ICT) software tools/models that are being utilized in the built environment, including three dimensional CAD modelling, were presented. In particular, Chau et al [24] presented a four dimensional visualization model designed to aid the knowledge and understanding of the relevance of modern computer graphics to the responsibilities of the construction site manager. The fact that the methods of drawings and graphs cannot be computationally analyzed even though they are capable of expressing complex concepts have spurred researchers to develop visual approaches and paradigms that can be used to capture and represent nodes in a computational form and which can be linked to and integrated with data sets and applications (e.g. [25]). One of the active research institutions in this area is the Pacific Northwest National Laboratory, USA [26].

The rest of the paper is arranged as follows: In Section II, some historical and contemporary ideas on mathematical modeling are presented. In Section III, the qualitative equivalence of ordinary differential equation (ODE) model of building structure is espoused. This is the main contribution in the paper. This model is an extension of an earlier application of the concept of qualitative equivalence of differential equations to fractals and computer networks [27, 28, 29]. Essentially, a similarity is established among fractals, networks and components of buildings (such as wall, roof, door etc.), namely that, all can take the form of a digraph. The focus is however on a door. Section IV discusses the model while Section V concludes the paper.

II REVIEW OF LITERATURE ON THE MATHEMATICAL MODELLING OF BUILDINGS

The use of mathematical modeling for studying physical structures has been an age long phenomenon. The first stage in the mathematical modeling process is the formulation of a real or analytical model of the (building or civil engineering) structure which is being studied [7, 17, 18]. Assumptions are then made to simplify the model by applying such physical laws as the stress-strain relationship for elastic beams, the Newton's laws, Lagrange's equation, strain-displacement relations etc. This is with a view towards formulating a mathematical problem for the physical problem.

Usually the mathematical problem would invariably be in the form of differential equation(s) of motion that describe the analytical model; generally, a continuous model leads to partial differential equations while a discrete-parameter model leads to ordinary differential equations. This is then followed by the solution of the resulting equations to obtain the dynamical response. The equations are solved subject to some specified initial conditions and inputs from external sources (such as dimensions, material properties, loads). The solution is thereafter interpreted in terms of the time history of the motion of the structure. This gives the predicted behaviour of the real structure otherwise called the response. In particular, development in mathematical modeling for building and civil engineering structures is intertwined with the evolution or history of structural mechanics (statics or dynamics) [3, 4, 30]. Table 1 presents a summary of historical development in mathematical modeling of these structures (structural mechanics) from the ancient/earliest times to modern times.

Table 1: Summary of Some Historical And Contemporary Developments In Mathematical Modelling Of Building And Civil Engineering Structures

S/N	Name of Scientist	Highlight of Contribution to Knowledge
1.	Imhotep (c. 2700 BC), Egyptian architect and structural engineer	He is credited with the construction of the first recorded step pyramid. He is thus often referred to as the father of structural engineering.
2.	Aristotle (384-322BC); Greek scientist and philosopher	He propounded the subject matter of structural statics.
3.	Leonardo da Vinci (1452-1519); Italian scientist and philosopher	He laid the foundation of mechanical science.
4.	Galileo Galilei (1564-1642); Italian scientist	He expounded theories of structural analysis including those for stresses in frames and strains in beams.
5.	Robert Hooke (1635-1703); English scientist	He discovered the basic law of elasticity. In the process of doing this, he corrected some of the theories of Galileo. He also explained the principle of the catenarian arch.
6.	Claude-Louis Navier (1785 - 1836), French physicist and engineer	He established the theory of elasticity. Among others, he conjectured Navier's hypothesis (1926) which states that 'the plane-sections remain plane', in agreement with an earlier work of Edme Mariotte (1620 - 1684).
7.	The Bernoulli brothers, Swiss scientists. They include (i) Jacob Bernoulli (1654-1705) (ii) Johann Bernoulli (1667-1748), Jacob's junior brother (iii) Daniel Bernoulli (1700-1782), the son of Johann.	They applied the principle of the infinitesimal calculus to calculate the deflections of beams. They also developed the principle of the catenarian arch.
8.	Leonard Euler (1707 – 1783), Swiss scientist	He applied the principle of the infinitesimal calculus to analyze the buckling loads of perfect structures.
9.	Gottfried Wilhelm Von Leibnitz (1646 – 1746), German scientist	He discovered the concept of infinitesimal calculus.
10.	Sir Isaac Newton (1642 – 1727), English scientist	He co-discovered infinitesimal calculus with Leibnitz.
11.	Thomas Young (1773 - 1829), English physician and polyvalent intellectual	He improved on the work of Euler with respect to imperfect and eccentrically loaded columns. This work inspired theories of elastic stability, which is the basis of design of modern bridges.
12.	Charles Bage (1751 - 1822), English architect	He built the first building in the world which used an interior iron frame
13.	William Strutt (1756 - 1830), English civil engineer and architect	He built a prototype fireproof mill which is essentially the first fireproof building.
14.	Carlos Alberto Castigliano (1847 - 1884), Italian scientist	He proved a theorem for computing displacement as partial derivative of the strain energy.
15.	Joseph Aspdin (1778–1855), English inventor	He obtained patent for Portland Cement.
16.	Joseph-Louis Lambot (1814-1887), French inventor	He constructed a rowing boat which was built of ferrocement, thus paving way for reinforced concrete.
17.	Henry Bessemer (1813-1898), English inventor	He developed the Bessemer process for producing steel.
18.	Vladimir Shukhov (1853-1939), Russian civil/structural engineer and architect	He developed analysis methods for new structural geometries such as tensile structures, thin-shell structures, lattice-shell structures and hyperboloid structures.
19.	Eugene Freyssinet (1879 – 1962), French civil/structural	He discovered a way of solving the challenges arising from weakness of concrete structures under tension. This led to his construction of 6 prestressed

	engineer	concrete bridges. His discovery has applications in such area as the design of airship hangars.
20.	Hardy Cross (1885 – 1959), American civil/structural engineer	He developed the moment distribution method. This enables reliable approximation of the real stresses of complex structures i.e. the method is useful for structural analysis of statically indeterminate structures.
21	John Fleetwood Baker (1901 – 1985), British scientist and civil/structural engineer	He developed the plasticity theory of structures thus paving way for reliable design of steel structures.
22.	Alexander Hrennikoff (1896 – 1984), Russian-Canadian structural/civil engineer	He discovered an approach to the discretization of plane elastic problems using lattice framework. This paved the way for finite element analysis, a numerical technique.
23.	Richard Courant (1888 – 1972), German-American scientist	He developed the mathematics of finite element analysis, which is nowadays accomplished using a computer. This subsequently led to formal development of the finite element method by J. Turner, R. W. Clough, H. C. Martin and L. J. Topp.

A. *Tits Building Model*

The Tits building, due to J. Tits, is an abstract algebraic representation or model of a real building structure [19, 31]. The model is based on mathematical concepts such as lattice theory, graph theory and general set theory. In particular, lattice theory is generally useful in providing a common abstract setting for studying subsystems of certain systems e.g. subgroups of a group [32]. Simply put, a lattice is a set T on which a relation ~ is refined such that this relation is reflexive, (i.e. a~a), antisymmetry (a~b, b~a ⇨ a=b) and transitive (a~b, b~c ⇨ a~c) and in which any two elements of T have a greatest lower bound and a least upper bound. Essentially, a Tits building refers to a pair C =(C, S) in which C is a complex and S a family of finite sub-complexes (otherwise called apartments) possessing four (4) basic properties namely [19]:

(i) The complex is thick.
(ii) Each of the apartments A_i is thin.
(iii) Any two elements of a complex belong to an apartment.
(iv) Given two apartments A_1, A_2 in which $\alpha, \beta \in A_1 \cap A_2$, there exists an isomorphism $\theta: A_1 \rightarrow A_2$ such that $\theta(w) = w$ for all $w \in \alpha C \cap \beta C$.

Two concepts in mathematics are said to be isomorphic if they are structurally identical.

A complex in itself refers to a semilattice $T = (T, \leq) = (T, \wedge)$ which has a minimum element zero (0) such that for all $x \in T$, $xT = \{y \in T: y \leq x\}$ is a finite Boolean lattice (^ means "and"). The minimal elements of $T\backslash\{0\}$ are called vertices while the maximum elements of T are called chambers. If $x \in T$, then the rank of x is the number of vertices in xT. The basic assumption is that all chambers are of the same rank d and every element of a semilattice is a chamber. The rank of a semilattice is defined to be d. Let x, $x^{/}$ be chambers and suppose T is connected (i.e. there exist chambers $x = x_0, x_1, x_2, ---, x_m = x^{/}$ such that $x_i \wedge x_{i+1}$ has rank d-1 for $0 \leq i \leq m-1$. A subcomplex refers to an ideal of T [32, 33]. A (sub) complex is said to be thick if every element of rank d-1 is less than at least three (3) chambers. It is said to be thin if every element of rank d-1 is less than exactly two (2) chambers.

B. *Bioinformatics Model*

One of the most explored areas in building models is the bioinformatics model which is based on evolutionary/genetic algorithm. This algorithm is premised on the fact that a computer program employs a large population of different inputs such that each specifies one way of performing some task. The program transforms a randomly generated input into a set of corresponding outputs, in such a way that the set is evaluated based on some predetermined criteria. At every stage, a program creates a new set of outputs which it once again evaluates in order to produce a new generation of inputs. The process is repeated several times until a satisfactory output is arrived at. Thus, the principle of 'survival of the fittest' holds since it is only the individuals that perform best go on to have 'offspring'. Two of the evolutionary algorithm-based programs are Inventor and Emergent Designer [20, 34, 35].

In the Inventor, each gene determines a specific structural element such that genes in four separate sections of a building's genome correspond to the following four different kinds of elements, namely bracing, beam, column and support footings. Also, every gene has a numeric value which encodes the type of structural element to be used such that there exists a 1-1 mapping of the genes to the building components e.g. if the value of a gene which describes a bracing is 0, nothing should be put in that position. Emergent Designer is a computer program which permits the simulation of structures that can be developed from a design embryo. Just like Inventor, the program also permits the simulation of structures which evolve through crossovers and mutations. The program allows the entire genome to evolve i.e. the program modifies both the design embryo and the design rule over many generations. The advantage of Emergent Designer over Inventor is that the former is stiffer (i.e. has a higher sway) under applied load such as the normal weight and the wind load. In general, a novel way of maintaining a reduced weight, especially in very tall buildings, is to use macro-diagonals. These are external cross-bracings which span large areas of a building. Examples of classic high rise buildings with embedded macro-diagonals include the Bank of China Tower, Hong Kong and the John Hancock Tower, Chicago, USA. A cursory look at the Swiss Re Tower, London shows that it is a classic example of modern day buildings which mimic nature (biomimicry), and so amenable to the principle of genetic algorithm. Specifically, the tower resembles a microorganism called glass sponge, and so a computer simulation may be carried out to create optimized designs using the biological processes of genetic crossover, mutation and evolution.

III QUALITATIVE EQUIVALENCE MODEL OF BUILDINGS

Many, if not majority, of real life phenomena are invariably modeled using differential equation. This is due to the fact that the equation describes the rate of change of a variable subject to another variable such as time i.e. dynamical phenomena. Generally, an equation in a variable that changes with respect to another variable may assume any of the following four generic types, namely, ordinary differential equation, partial differential equation, difference equation and differential-difference equation. The type of equation to be used for modeling depends on the nature of the problem to be solved. The qualitative theory of ordinary differential equations is a novel theory which attempts to make prediction about the changes in structural or dynamical systems by observing patterns instead of explicitly solving any equation, in contrast to the analytic method i.e. it studies the geometry of the solution curves of differential equations instead of solving them explicitly by using conventional methods such as the use of specific formulas, such as the popular integration by parts, separation of variables, as well as the series solution method [36, 37, 38].

This section presents an exposition on the use of the qualitative theory of differential equations to model the basic mathematical geometries of lines, planes and their derived shapes. Specific connection with building structures is then made. Examples of differential equation-based models of building structure which are non-qualitative are [39, 40]. Two general models based on the idea of qualitative equivalence of differential equations are [41, 42]. In particular, in [42], the authors presented a review on qualitative reasoning such as learning qualitative differential equation model and qualitative simulation. Applications of these models in physics, biology and medical science are then highlighted. In [41], the author presented concept of equivalence by considering whether the reflecting functions of two differential systems are coincident in their common domain. If they are, the systems are defined to be equivalent.

The present model is based on the ordinary differential equation

$$x' = f(x) \qquad (3.1)$$

called first order autonomous equation, where $f(x) = \Sigma a_i x^{n-i}$, $0 \le i \le n$, is the general polynomial of degree n. The equilibrium (or critical) points of the equation are those values of n which would make the right hand side of the equation to be zero e.g. the equation $x' = x^2 - 9$ has two equilibrium points namely $x = \underline{+}\, 3$. Of particular importance in the model is the geometrical representation of the qualitative behaviour of (3.1) called the phase portrait or phase diagram. This diagram is completely determined by the nature of the equilibrium points of (3.1).

A. *Algorithm*

The algorithm of this model is as follows:

Procedure *QUALITATIVE EQUIVALENCE MODEL*
Step 1: Start
Step 2: Consider a set of first order autonomous ODEs of the form

$$\{x' = f(x) = \sum a_i x^{n-i},\ a_i \in \mathbb{R},\ 0 \le i \le n, \text{ for fixed } n\} \qquad (3.2)$$

where R is the set of real numbers, and n a natural number.

Step 3: For a given n, consider all the possible equilibrium points (ep).

(i) If n = 1, there exists only one ep which is necessarily real.
(ii) If $n \ge 2$, all the ep may be real or some may be complex, occurring as complex conjugates.

Step 4: For a given n in Step 3 above, construct the various phase portraits based on the ep.

Step 5: For two ordinary differential equations D_1, $D_2 \in (3.2)$, define an equivalence relation ~ on (3.2) as $D_1 \sim D_2$ if D_1 and D_2 have the same geometrical shape, otherwise called phase portraits.

Step 6: Based on the Fundamental Theorem of Equivalence Relations [32], (3.2) can be classified into disjoint equivalence classes, referred to as the qualitative classes in the qualitative theory of differential equations.

Step 7: Study the physical structure of a building component to be modeled.

Step 8: Superimpose two or more combinations of phase portraits to get the shape of the desired building component.

Step 9: Stop

B. *The Case n = 1 and n = 4*

The cases in which n = 1 and n = 4 in (3.2) are classic examples of cases which enable a door or door frame to be constructed. The former is the simplest case, where f(x) is linear [35]. These cases are thus considered in this subsection. When n = 1, the four basic phase diagrams are as shown in Figure 1, where c is the equilibrium point.

In Figure 1(a), f(x) decreases when c > 0 and increases when c < 0. In Fig. 1(b), f(x) increases when c > 0 and decreases when c < 0. In the case of Fig. 1(c), f(x) decreases when c > 0 and also increases when c < 0 while in Fig. 1(d), f(x) decreases when c > 0 and also decreases when c < 0. This form the set:

S = {Attractor (A), Repellor (R), Positive Shunt (P), Negative Shunt(N)} (3.3)

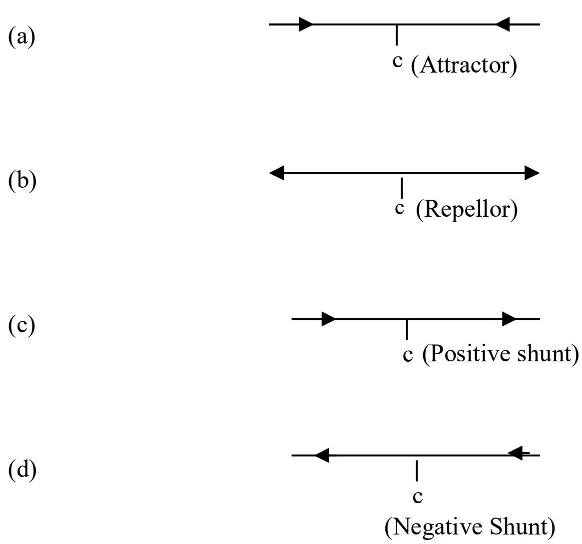

Figure 1: Basic digraphs (phase portraits) when f(x) is linear

If the direction of each of the elements of this set is ignored, then a superimposition by end points of the elements models such building components as louvres, doors, floors and roofs. As the degree n of f(x) increases and when some or all the equilibrium points of (3.1) have complex values, more interesting phase diagrams are formed using (3.2). When disconnected straight lines are formed, the end points of these lines are joined to form a single structure. The set defined in (3.2) may therefore be partitioned into disjoint equivalent or qualitative classes such that all equations having the same phase diagram belong to the same class.

Similarly, consider the case in which n = 4 [28]. This gives rise to the equation.

$$x^{/} = ax^4 + bx^3 + cx^2 + dx + e \qquad (3.4)$$

where a, b, c, d, e are real numbers, $a \neq 0$. (3.2) then has (i) four real equilibrium points (ii) four complex equilibrium points (comprising of two complex conjugates) as well as (iii) two real and two complex equilibrium points. In case (i), there are 21 possible subsets of (3.2) with 18 actual qualitative classes i.e. only 18 distinct digraphs exist, each of which is a directed straight line graph. The direction of each of the diagrams may be inferred by considering the set {ARAR, RARA, ARPP, RANN, AR, RA, PAR, NRA, ANR, RPA, PPP, NNN, ARP, RAN, P, N, PP, NN}. In case (ii), the two basic graphs are:

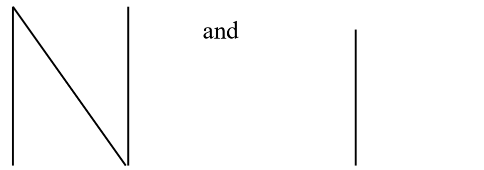

Figure 2(a) Figure 2(b)

Figure 2: Basic graphs when all equilibrium points are complex for n = 4

The three basic graphs in case (iii) are:

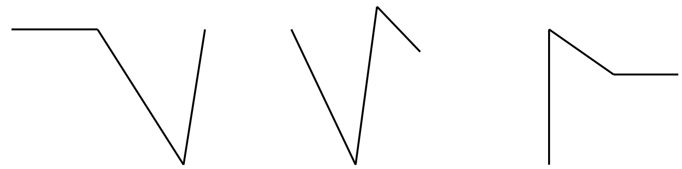

Figure 3(a) Figure 3(b) Figure 3(c)

Figure 3: Basic graphs when two of the equilibrium points are complex for n = 4.

C. *Illustration: Construction of a Door*

Consider a door or a door frame as a building component being modeled. A typical three dimensional shape of the component is as shown in Figure 4. One part of the figure shows the door in an open position, clearly revealing its frame. The second part shows the door in its closed position. Structurally, the door consists of four (4) nodes and four (4) edges. If the direction in Figure 1 is ignored and the node of one diagram rests on the node of another, then a rectangle having 4 nodes and 4 edges result. By stretching the sides of the figure to the appropriate value of length and breadth, then a door or door frame is constructed. Alternatively, consider Figure 2(b), which consists of two (2) vertical lines, one on the left hand side (LHS) and the other on the right hand side (RHS). Suppose another copy of the figure is produced and is rotated 90^0 counterclockwise. By superimposing it on top of the original figure, such that the line on its LHS now rests on the bottom of the original figure while the line on its RHS lies on the bottom of the original figure, such that the nodes are all joined together, then a door frame structure is formed.

Figure 4: A door [43]

IV DISCUSSION

In general, every component of a building can be aptly modeled using the idea of the qualitative equivalence (or theory) of ordinary differential equations. In (3.2), the case in which the value n results into complex conjugates is more interesting as it gives rise to more robust structures. The present paper generates k-state systems (where $k \geq 1$), such that the actual value of k depends on the degree n in (3.2) such that $k \leq n$. The quaternary or 4-state codes, in which $k = 4$, arising from the present paper can be reduced to the binary or 2-state codes designed and analyzed in [7] by considering Figure 1. In the figure, if phase portraits having arrows moving in the same direction are considered to be isomorphically the same, then 1(a) and 1(b) describes just a single state AR while 1(c) and 1(d) describes another state PN. Thus, the 4-state system {A, R, P, N} reduces to a 2-state system {AR, PN}, which is equivalent to {0, 1}. The idea in the paper can be integrated into existing AUTOCAD related software to bring about working drawings (including plans, sections and elevations) produced by orthographic, isometric, axonometric or pictorial projections [2]. It can also be improved and developed as a stand-alone design software, such as is the case with the software Inventor and Emergent Designer [20].

V CONCLUSION

This paper has presented a discrete computing approach to modeling components of building structures, with emphasis on the use of differential equations. The idea relates to the modern theory of differential equations otherwise known as the qualitative theory. It can be developed, via simulation, into a distinct computer application software that can be adapted to or integrated into existing computer-aided design or graphics software such as AUTOCAD. This would ease the work of architects and building professionals with respect to effective and efficient design and construction of modern building structures. The idea is capable of assisting in producing working drawings of simple and complex building components and elements such as the trussed or framed roofs. For instance, in the king post and the queen post timber trusses, the principal rafter, strut, tie beam, straining beam, king post and queen posts are formed from the shapes which arise when the critical points of (3.2) are a mixture of real and complex values for a high value of n. The paper can be viewed as a contribution to information technology in building technology, and also to discrete structures of computer science (DSCS). Essentially, the paper has established a connection between building (civil) engineering and computer science. That is, it is shown that design of building structures can be accomplished via the instrumentality of DSCS. Similar application can be extended to the design of roads, bridges and rail tracks.

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