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On the Log-Gamma Exponentiated Weibull Regression Model: An Application to Recurrence of Bank Failure in Nigeria

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Abstract
In this paper, the log-gamma exponentiated Weibull distribution, a modified version of exponentiated Weibull distribution was developed with a view to model recurrence of bank failure in Nigeria. The new distribution is a generalization of the Weibull, Exponentiated Weibull and Exponentiated exponential distribution. The statistical properties of the new distribution are discussed. These include the moments, moment generating function, binomial series expansion and distribution of the order statistics. The proposed distribution will be a very useful alternative to well known distribution for failure time data analysis. A regression model based on the new distribution to predict recurrence of bank failure (after recent bank reform in Nigeria) taking into consideration certain bank characteristics as covariates was also constructed.

Key words: Exponentiated Weibull, Censored data, Gamma exponentiated Weibull Log-gamma Exponentiated Weibull, Survival function.

Introduction

Bank failure occurs when a bank is unable to meet its obligations to its depositors or other creditors because it has become insolvent or too illiquid to meet its liabilities. More specifically, a bank usually fails economically when the market value of its assets declines to a value less than that of its liabilities. Consequently, the insolvent bank either borrows from other solvent banks or sells its assets at a lower price than its market value so as to generate liquid money to pay off its depositors on demand. The inability of the solvent banks to lend liquid money to such insolvent bank creates a bank panic among its depositors, as more depositors try to withdraw cash deposits from the bank. As such, the bank is unable to fulfill the demands of all of its depositors on time. Also, a bank may be taken over by the regulating government agency, if shareholders equity (i.e. capital ratios) is below the regulatory minimum. The failure of a bank is generally considered to be of more importance than the failure of other types of business firms because of the inter-connecting and fragility of banking institutions.

Research has shown that the market value of customers of the failed banks is adversely affected at the date of the failure announcements. It is often feared that the spillover effects of a failure of one bank can quickly spread throughout the economy and possibly result in the failure of other banks, whether or not those banks were solvent at the time as the marginal depositors try to withdraw cash deposits from these banks so as to avoid suffering losses. The spillover effect of bank panic or systemic risk has a multiplier effect on all banks and financial institutions leading to a greater effect of bank failure in the economy. As a result, banking institutions are typically subjected to rigorous regulation, since bank failures are of major public concern in countries across the world.

Recent research offers important insights into how liquidity risk causes or exacerbates financial crises [5]. Particularly important are the works on short-term debt rollover risk, which suggest the potential predictive power of debt market liquidity risk for corporate default [13], and underscore the systemic nature of rollover risk and the link between rollover risk

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and market freeze [1]. Furthermore, market illiquidity can lead to the insolvency of financial firms through the market valuation channel [3,4] and [17].

The increasing emergence of studies on mortality of business institution using financial indicators occurred after the performance of the dichotomous classification and the use of discriminant function. Statisticians started by using discriminant analysis for failure prediction model for banks. Discriminant analysis is however restrictive because of its assumption of normality of data. Other techniques adopted to constructing failure prediction models for banks are logistic regression analysis and survival analysis.

The list of contributors to the development of models in the recent past for predicting bank failure includes [2], who had preference for discriminant analysis [19] preferred logistic regression as their guiding principle, while [1] strongly believed in the use of Cox proportional hazard model for analysis of bank failure. These distributions are generally applied in biological and medical sciences to predict the survival time of patient's, who have been subjected to an initial event (e.g a surgical operation) but are now being used to study failure of financial institutions because of their operating characteristics.

Bank Failure in Nigeria

In spite of the 1952 Banking Ordinance, the Nigerian economy has experienced a number of bank failures. The period 1994-2003 also witnessed a wave of system distress culminating in another round of bank failures. Following the heavy impact this ugly and recurring development has inflicted on the Nigeria Economy, the 2004 banking sector reforms swept away 14 banks.

The occurrence of bank failure in the country therefore has generated utmost concern not only to the practitioners and the academia but also to the entire nation in general [2, 4]. The "mortality" pattern of financial entities in Nigeria is very interesting to study using duration models, which can be helpful tools in explaining and predicting not only the "death" probability, but also how this probability develops over time, and determine what economic variables are affected when the potential failure occurs. In this paper, we propose a location-scale regression model based on the log-Gamma exponentiated Weibull distribution referred to as the log-Gamma exponentiated Weibull regression model which is a feasible alternative for modeling the type of recurrence of bank failure in Nigeria.

The Gamma-Exponentiated Weibull Family of Distribution

In recent time, compounded Weibull distribution have been developed with a view to providing better representation of certain data sets than the traditional Weibull models. The exponentiated Weibull distribution developed by [15] was found to be superior to traditional Weibull model due to its flexibility. Another generalization of the Weibull distribution is the beta-Weibull model proposed by [11] and further investigated by [7] and [8]. They discussed some properties of the four parameter beta-Weibull distribution. The distribution is applied to censored data-sets on bus-motor failures, a censored data set on head-and-neck cancer clinical trial and also to survival data. A researcher [9] extended the Weibull distribution while [10] worked on the generalized form of Kumaraswamy Weibull distribution and applied it to failure rate data. Jones in [14] studied a family of distribution that arises naturally from the distribution of order statistics. The beta-generated family proposed by [14] and further studied by Jones was discussed in [18], who proposed an alternative gamma-generated family. Different authors have used both beta-generated family and gamma generated family to develop a new vibrant distribution with better data representation especially in survival analysis.

Jones gave the generalized beta-generated family, beta generated link function in [14] as

$$g(x)=\frac{cf(x)\left[F(x)\right]^{a-1}\left[1-F(x)^{c}\right]^{b-1}}{\beta(a,b)} \quad ...(1)$$

while [11] assumed the exponential density function defined by

$$f(x)=\frac{K}{\lambda}\left(\frac{x}{\lambda}\right)^{K-1}\ell^{-\left(\frac{x}{\lambda}\right)^{K}},$$

as the baseline continuous distribution and considering the cumulative distribution $F(x)$ function of the Weibull distribution defined as

$$F(x)=\left[1-\ell^{-\left(\frac{x}{\lambda}\right)^{K}}\right]$$

[11] employed equation (1) with C =1, to obtain Beta-Weibull Distribution defined by

$$g(x)=\frac{K\ x^{K-1}}{\lambda^{K}\ \beta(a,b)}\ \ell^{-b\left(\frac{x}{\lambda}\right)^{K}}\left[1-\ell^{-\left(\frac{x}{\lambda}\right)^{K}}\right]^{a-1} \quad \ldots(2)$$

where $a,\ b,\ \lambda,\ K$ *and* $x>0$

By letting $a=b=1$ in (1) and using the Weibull distribution $F(x)$, [15] obtained a distribution whose random variable is distributed as exponentiated Weibull defined by

$$K(x)=\frac{C\,Kx^{K-1}}{\lambda^{K}}\ \ell^{-\left(\frac{x}{\lambda}\right)^{K}}\left[1-\ell^{-\left(\frac{x}{\lambda}\right)^{K}}\right]^{c-1} \quad \ldots(3)$$

where $a,\ b,\ \lambda,$ *and* $x>0$

The gamma generated family of distributions developed by [18] is defined by

$$g(x)=\frac{1}{\Gamma(\delta)}\left[-\log F(x)\right]^{\delta-1} f(x) \quad \ldots(4)$$

(where, δ is the shape parameter of the gamma-generated family of distribution) to develop a new distribution called the Gamma-exponentiated Weibull distribution using the exponentiated Weibull $F(x)$ and $f(x)$ defined by

$$F(x)=\left[1-\ell^{-\left(\frac{x}{\lambda}\right)^{K}}\right]^{\alpha} \quad \ldots(5)$$

$$f(x)=\frac{K\alpha}{\lambda}\left(\frac{x}{\lambda}\right)^{K-1}\ell^{-\left(\frac{x}{\lambda}\right)^{K}}\left[1-\ell^{-\left(\frac{x}{\lambda}\right)^{K}}\right]^{\alpha-1} \quad \ldots(6)$$

(see [16])

By putting equations (5) and (6) in (4), [18] obtained:

$$g(x)=\frac{K\alpha^{\delta}}{\lambda\Gamma(\delta)}\left(\frac{x}{\lambda}\right)^{K-1}\ell^{-\left(\frac{x}{\lambda}\right)^{K}}\left[1-\ell^{-\left(\frac{x}{\lambda}\right)^{K}}\right]^{\alpha-1}\left[-\log\left(1-\ell^{-\left(\frac{x}{\lambda}\right)^{K}}\right)\right]^{\delta-1} \quad \ldots(7)$$

with corresponding survival function defined by,

$$S(x)=\frac{1}{\Gamma(\delta)}\int_{0}^{-\log\left[1-\ell^{-\left(\frac{x}{\lambda}\right)^{K}}\right]^{\alpha}} t^{\delta-1}\ell^{-t^{K}}\,dt$$

and $G(x)=1-\frac{1}{\Gamma(\delta)}\int_{0}^{-\log\left[1-\ell^{-\left(\frac{x}{\lambda}\right)^{K}}\right]^{\alpha}} t^{\delta-1}\ell^{-t^{K}}\,dt$,

Thus,

$$G(x)=1-\frac{\gamma\left[-\log\left(1-\ell^{-\left(\frac{x}{\lambda}\right)^{k}}\right)^{\alpha},\delta\right]}{\Gamma(\delta)} \quad \ldots(8)$$

is the cumulative distribution function

where $\delta, K, \alpha,\ \lambda>0$ and

$\gamma(x,\delta)=\int_{0}^{x}x^{\delta-1}\ell^{-x}dx$ is the incomplete gamma function.

The Log-gamma Exponentiated Weibull Distribution

For the first time, we propose a log-gamma exponentiated Weibull (LGEW) regression model to predict the *t* month failure recurrence free probability after bank recapitalization and banking reform in Nigeria. We start by introducing a log-gamma exponentiated Weibull distribution which will subsequently be converted to a regression model to achieve the intention of the study.

Let T be a random variable having the gamma exponentiated Weibull distribution in equation (7), by defining another random variable $Y = \log(T)$ the density function of Y can be expressed as

$$f(y^{'}, \alpha, \delta, \mu, \delta) = \frac{\alpha^{\delta}}{\delta\Gamma(\delta)} \ell^{\left(\frac{y-\mu}{\delta} - \ell^{\frac{y-\mu}{\delta}}\right)} \left(1 - \ell^{\frac{y-\mu}{\delta}}\right)^{\alpha-1} \left[-\log\left(1 - 1 - \ell^{\frac{y-\mu}{\delta}}\right)\right]^{\delta-1} \ldots(9)$$

where, $\delta = K^{-1}$ *and* $\mu = \log(\lambda)$ *and* $-\infty < \mu,\ y < \mu < \infty$ *and* $\delta > 0$

The new model (9) is referred to as the log-gamma exponentiated Weibull distribution (LGEW) distribution. (i.e $Y \approx LGEW(\mu, \sigma, \alpha, \delta),$) where μ is a location parameter, σ is a dispersion parameter and α *and* δ are shape parameters.

Properties of the LGEW

Here, we study some properties of the LGEW. Let us define the standardized random variable by $Z = \frac{y-\mu}{\sigma}$ from whence the density function of Z is derived using the fact that

$$f(x) = \varphi(y(z))\,|J| \qquad where$$

$$|J| = \frac{1}{\frac{dZ}{dy}} = \sigma$$ so that,

$$f(z) = \frac{\alpha^{\delta}}{\Gamma(\delta)} \exp\left[Z - \exp(Z)\right]\left[1 - \exp(Z)\right]^{\alpha-1}\left[-\log(1 - \exp(z)\right]^{\partial-1} \qquad -\infty < z < \infty \tag{10}$$

Binomial Series Expansion

We can also use the binomial series expansion to express the log-Gamma exponentiated Weibull distribution using the fact that

$$(1-m)^{b} = \sum_{j=0}^{\infty} (-1)^{j} \binom{b}{j} m^{j}$$

Thus equation (10) can be written as

$$f(z) = \frac{\alpha^{\delta}}{\Gamma(\delta)} \sum_{j=0}^{\infty} (-1)^{j} \binom{\alpha-1}{j} \exp\left[Z(1+j) - \exp(Z)\right]\left[-\log\left(1 - \exp(Z)\right)\right]^{\delta-1} \tag{11}$$

Also, using the power series

$$-\log(1-y) = \sum_{i=0}^{\infty} \frac{y^{i+1}}{i+1}, \qquad to\,obtain$$

$$f(z) = \frac{\alpha^{\delta}}{\Gamma(\delta)} \sum_{j=0}^{\infty} (-1)^{j} \binom{\alpha-1}{j} \exp\left[z(1+j) - \exp(z)\right]\left[\sum_{i=0}^{\infty} \frac{\exp\left(z(i+1)\right)}{1+i}\right]^{\delta-1} \tag{12}$$

Moments and Moment Generating Function

Following [8], who derived the moment of the Exponentiated Weibull distribution, the r^{th} moment about the origin of the log gamma exponentiated Weibull distribution is derived, using (12) to get

$$\mu_r^1 = E(z^r) = \int_{-\infty}^{\infty} z^r \frac{\alpha^\delta}{\Gamma(\delta)} \sum_{j=0}^{\infty} (-1)^j \binom{\alpha-1}{j} \exp\left[z(1+j) - \exp(z)\right] \left[\sum_{i=0}^{\infty} \frac{\exp[z(i+1)]}{1+i}\right]^{\delta-1} dz$$

set $P = \exp(z)$ *i.e* $z = \log P$; we obtain

$$\frac{dz}{dp} = \frac{1}{p} \Rightarrow dz = \frac{dp}{p}$$

$$\mu_r^1 = \frac{\alpha^\delta}{\Gamma(\delta)} \sum_{\alpha}^{\infty} (\log p)^r \sum_{j=0}^{\infty} (-1)^j \binom{\alpha-1}{j} p^{1+j} \exp(-p) \frac{dp}{p} \left[\sum_{i=0}^{\infty} \frac{\exp(\log p^{1+i})}{1+i}\right]^{\delta-1}$$

$$\mu_r^1 = \frac{\alpha^\delta}{\Gamma(\delta)} \sum_{\alpha}^{\infty} (\log p)^r \sum_{j=0}^{\infty} (-1)^j \binom{\alpha-1}{j} p^j \exp(-p) \left[\sum_{i=0}^{\infty} \frac{p^{1+i}}{1+i}\right]^{\delta-1} dp \tag{13}$$

Equation (13) is the r^{th} moment of density function in (10) while the corresponding moment generating function is

$$M_Z(t) = \exp(tz)$$

$$= \frac{\alpha^\delta}{\Gamma(\delta)} \int_0^\infty 2 \sum_{\alpha}^{\infty} \sum_{j=0}^{\infty} (-1)^j \binom{\alpha-1}{j} p^{j+t} \exp(-p) \left[\sum_{i=0}^{\infty} \frac{p^{1+i}}{1+i}\right]^{\delta-1} dp$$

$$M_Z(t) = 2 \frac{\alpha^\delta}{\Gamma(\delta)} \sum_{j=0}^{\infty} (-1`)^j \binom{\alpha-1}{j} \Gamma(j+i+1) \left[\sum_{i=0}^{\infty} \frac{p^{1+i}}{1+i}\right]^{\delta-1} \tag{14}$$

Order Statistics

In many areas of statistical theory and practice, order statistics play a predominant role particularly in reliability study, most especially in engineering.

We now give the density of the *rth* order statistics $Z_r; n, f_r : n(z)$ say, in a random sample of size n from the log-Gamma exponentiated Weibull. It is well known that the r^th^ order statistics z_r is

$$f_r : n(Z) = \frac{1}{\beta(r, n-r+1)} f(z) F(z)^{r-1} \left[1 - F(z)\right]^{n-r} \tag{15}$$

$for\ r = 1, \ldots, n$

The cumulative distribution of $F(z)$ of the distribution in (10) is obtained using $F(z) = 1 - S(z)$ where

$$S(z) = \frac{\gamma\left[-\log(1-\exp(Z))^\alpha, \delta\right]}{\Gamma(\delta)} \tag{16}$$

Using (10) and (16) in (15), we have

$$f_{r:n}\underset{\Gamma(\delta)}{(z)} = \frac{\alpha^{\delta}}{\beta(r, n-r+1)}\left[\frac{\gamma\left[-\log\left(1-\exp(z)^{\alpha}, \delta\right)\right]}{\Gamma(\delta)}\right]^{n-r}\left[1-\frac{\gamma\left[-\log\left(1-\exp(z)^{\alpha}, \delta\right)\right]}{\Gamma(\delta)}\right]^{r-1}$$

$$\exp\left(z-\exp(z)\right)\left[1-\exp(z)\right]^{\alpha-1}\left[-\log\left(1-\exp(z)\right)\right]^{\delta-1} \qquad (17)$$

The LEGW Regression Model

In many practical applications, lifetimes are affected by covariates such as blood pressure, cholesterol level among others. The covariate vector is denoted by $x = (x_1, x_2, ...,, x_p)^T$. which is related to the responses variable $Y = \log T$ through a regression model.

Based on the log-Gamma exponentiated Weibull density function, we propose a linear location-scale regression model relating to the response variable y_i and the explanatory variable vector $X_i^T = (x_1, x_2, ...,, x_p)$ as follows

$$y_i = X_i^T \beta + \sigma Z_i,\; i = 1,..., n \qquad \dots (18)$$

where, Z_i denotes the random error distributed according to density function in (9), and $\beta = (\beta_1,..., \beta_p)^T$, $\sigma, \alpha, \delta > 0$ are parameters to be estimated. The location of the response variable y_i is $X_i^T \beta$ i.e. $\mu_i = X_i^T \beta$ for $i = 1,..., n.$ The LGEW regression model in (18) presents new development in fitting many different types of similar data. It contains as special sub-models well known in the literature. For $\delta = 1,$ it becomes log-exponentiated Weibull regression if in addition $\sigma = 1$ we have log-exponentiated exponential regression model. By considering a sample $(y_i, x_1), ..., (y_n, x_n)$ of *n* independent observations, where each random response is y and explanatory variable. The log-likelihood function for the vector of parameter $\theta = (\alpha, \delta, \sigma, \beta^T)^T$ model (18) is defined as

$$l(\theta) = r\delta\log\alpha - 2r\log\Gamma(\delta) + \sum_{i=1}^{n}\exp\left(Z_i - \exp(z_i)\right) + (\alpha-1)\sum\log\left(1-\exp(z_i)\right) + (\partial - 1)$$

$$\sum\log(-\log(1-\exp(z_i))) + \sum\log\gamma\left[-\log(1-\exp(z)), \partial\right]$$

where, r is the number of uncensored observations (failures) and $z_i = \left(\frac{y_i - X_i^T\beta}{\delta}\right)$, the standardized response variable.

The maximum likelihood estimate of the vector θ of unknown parameters can be obtained by maximizing the log-likelihood function in (19).The statistical software such as Subroutine NL Mixed in SAS, R and S-plus can be used to implement the above result.

Application

The log-gamma exponentiated Weibull distribution (LGEW) distribution can be applied by fitting the model in (19) to the data of a given sample of banks. The analysis can be conducted through Accelerated Failure Time (AFT) Model. It is also possible to analyze the effect of covariant on survival time, the new model being in the class of parametric models allows us to check how closely the model fits the data. Unlike semi-parametric proportional hazard models (such as Cox model), it has the advantage of allowing the explicitly, test the shape of the hazard function.

The estimated model in (14) can take the form

$$\log(T) = \beta_0 + \beta_1(CAP_i) + \beta_2(MUTUAL_i) + \beta_3(Management)_i + \beta_4(Balance\ \ Sheet)_i + \beta_5\ dummyYear_n + \sigma z_i$$

where, T is the time until the defiant bank I, CAP– the log of capital at the start-up, MUTUAL is a dummy variable identifying mutual banks Management, market and balance-sheet are vectors of bank and market-specific regressions. The shape parameter σ allows for different distributions of z besides the normal distribution i.e. log-gamma exponentiated Weibull distribution.

After estimating the parameters in the model, the survival function in (12) can be used to predict the probability of recurrence of bank failure after the bank reforms in Nigeria.

Conclusion

The log-gamma exponentiated Weibull (LGEW) distribution was developed with a view to modelling bank failure in Nigeria. Attempts have been made to derive expression for its moment generating function, moment survival function and order statistics. The hazard rate function can accommodates varying degree of shapes.

Based on this new distribution, we also propose a LGEW regression model which is found appropriate for modelling survival data particularly bank failure data. The new regression model serves as a good alternative for lifetime data analysis. It has flexibility advantage over its sub-models such as log exponentiated Weibull, log exponentiated exponential and log Weibull regression models.

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