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Panel Data Estimators in the Presence of Quadratic and Exponential Functional Forms of Heteroscedasticity

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Abstract
The problem of heteroscedasticity in panel data has been widely discussed in literature and has continued to attract the attention of researchers particularly in applied econometrics. In this study, we explore two functional forms of heteroscedasticity: Quadratic Heteroscedasticity Functional Form (QHFF) and Exponential Heteroscedasticity Functional Form (EHFF) in a random error component model. We use one-way error component model to evaluate these two forms of heteroscedasticity on individual effect of one error components model. In this paper, we design a Monte Carlo experiment to investigate the relative sensitivity of the following estimators: Pooled Ordinary Least Square (POLS), Between Group (BG), Within Group (WG) and Panel Generalize Least Square (PGLS) estimators, in the presence QHFF and EHFF on individual effect. The Monte Carlo experiments follow closely that of [1] and [2]. Using purposedly cross-sectional units, $N = 10, 30, 50$ and time periods $T = 5, 15, 20$ and replication of 2500 for various combinations of N and T dataset were generated. R Version 2.15.2 Statistical software is used for our analyses. The relative performances of these estimators were assessed using Bias (BIAS) and Root Mean Squared Error (RMSE). The estimators were then ranked according to their performances. The performance of estimators in the presence QHFF and EHFF were investigated under the finite sampling properties of Bias and RMSE, for the two experiments set up and estimators were ranked as follows in ascending order of their performances: PGLS, BG, WG and POLS. This result will helps in the choice of estimator in empirical work when there is presence of heteroscedasticity.

Key words: Panel data model, Quadratic heteroscedasticity functional form, Exponential hetero-scedasticity functional form, Estimators, Experiment.

Introduction

Different types of data are generally available for empirical analysis, namely, time series, cross section and panel. A data set containing observations on a single phenomenon observed over multiple time periods is called time series (e.g., GDP for several quarters or years). In time series data, both the value and the ordering of data points have meaning. In cross-section data, values of one or more variables are collected for several sample units, or entities, at the same point in time (e.g., crime rates for 50 states in the United States for a given year).

As largely acknowledged, heteroscedasticity is endemic when working with micro-economic cross-section data. A primary and well-known source of heteroscedasticity stems from differences in the size characteristic. On the other hand, the error variance may also systematically vary across observations of similar size. For example, the variance of firms profits might depend upon product diversification or research and development expenditures. Likewise, the variance of firms outputs might depend upon their capitalistic intensity and so on. Note that in practice, these different sources of heteroscedasticity may be simultaneously present.

Obviously, there is no reason to expect the heteroscedasticity problems associated with microeconomic panel data to be markedly different from those encountered in work with cross-section data. Nonetheless, the issue of heteoscedasticity received somewhat less attention in the literature related to panel data error components models than in the literature related to cross-section models. [3] seems to be the first to

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deal with the problem of heteroscedasticity in panel data. The study looked at heteroscedasticity and stratification in two-way error component models. The study involved spectral decomposition of the variance-covariance matrix to derive statistically efficient and computationally simple estimation procedures.

Both [3] and [4] were concerned with the estimation of a model allowing for heteroscedasticity on the individual-specific error term, i.e., assuming that $\mu_i \sim (0, \sigma^2_{\mu_i})$ while $v_{it} \sim IID(0, \sigma^2_v)$. In contrast, [5], [6], [7] and [8] adopted a symmetrically opposite specification allowing for heteroscedasticity on the remainder error term, i.e., assuming that $\mu_i \sim IID(0, \sigma^2_\mu)$ while $v_{it} \sim (0, \sigma^2_{v_i})$ [9] suggested an adaptive estimation procedure for an error component model allowing for heteroscedasticity of unknown form on the remainder error term, i.e., assuming that $\mu_{it} \sim IID(0, \sigma^2_\mu)$ while $v_{it} \sim IID(0, \sigma^2_{v_{it}})$, where $\sigma^2_{v_{it}}$ is a non-parametric function $f(z'_{it})$ of a vector of exogenous variables. They also suggested a robust version of the [10] LM test for no random individual effects, $\sigma^2_\mu = 0$, by allowing for adaptive heteroscedasticity of unknown form on the remainder error term.

In this paper, we focus on balanced micropanels with N large and T small. We want to evaluate two different heteroscedasticity functional forms when they are incorporated on individual effect of error component model. Also, we want to estimate and rank the performances of the following four estimators: Pooled OLS (POLS), Between Group (BG), Within Group (WG) and Panel GLS (PGLS). This paper is structured as follow: Section 2 presents the theoretical framework. Section 3 describes the data generating scheme. Section 4 presents results and discussion while the last section concludes the paper.

Theoretical Framework

A typical static panel data regression can be expressed as:

$$Y_{it} = \beta_o + \sum_{k=1}^{j} \beta_k X_{kit} + u_{it}; \quad i = 1, ..., N, \quad t = 1, ..., T \tag{1}$$

where, Y_{it} is the dependent variable and X_{kit} are the matrix of explanatory variables. The subscripts i and t as earlier defined refer to cross-sectional and time series dimensions respectively. u_{it} is the composite error term which can be decomposed further into specific effects and remainder disturbance term.

There are two sets of specific effects namely the individual specific effects and time specific effects. If only one set of specific effects is included in the regression, such is referred to as one-way error components model. However, if both sets of specific effects are included, we refer to the model as two-way error components model. Equations (2), (3) and (4) show decomposition of u_{it} into one-way and two-way error components.

$$u_{it} = \mu_i + v_{it} \tag{2}$$

$$u_{it} = \lambda_t + v_{it} \tag{3}$$

$$u_{it} = \mu_i + \lambda_t + v_{it} \tag{4}$$

where, μ_i and λ_t denote the observed individual and time specific effects respectively.

For this paper, we shall limit our empirical applications to the one-way error components as stated below.

$$\begin{aligned} y_{it} &= \beta_0 + x_{it}\beta_k + u_{it} \\ u_{it} &= \mu_i + v_{it}, \\ i &= 1, ..., N, \qquad t = 1, ..., T \end{aligned} \tag{5}$$

where,

y_{it} is the dependent variable,

x_{it} is $(1 \times k)$ vector of explanatory variable

β_k is $(k \times 1)$ vector of coefficients

μ_i represents unobserved cross-sectional (individual) effects for N cross sections,

v_{it} represents remainder disturbance term.

The index i refers to the individuals and the index t to the observations of each individual. The total number of observations is NT. The error terms μ_i and v_{it} are assumed mutually independent and normally distributed according to:

$$v_{it} \sim N(0, \sigma_v^2); \quad \mu_i \sim N(0, \sigma_{\mu i}^2);$$

$$\sigma_\mu^2 = \sigma_\mu^2 h_\mu(f_i^{'}\theta), i = 1, \ldots, N, t = 1, \ldots, T$$

where, $h_v(.)$ and $h_\mu(.)$ are arbitrary non-indexed (strictly) positive twice continuously differentiable functions satisfying $h_v(.) > 0, h_\mu(.) > 0, h_v(0) = 1, h_v = (0) = 1$ and $h_v^{(1)}(0) \neq 0, h_\mu^{(1)}(0) \neq 0,$ where $h_\mu^{(1)}(x)$ denotes the first derivative of $h_\mu(x)$ with respect to $x.z_{it}$ and $f_i^{'}$ are respectively $(k_\theta \times 1)$ is a vector of strictly exogenous regressors while θ is a $(1 \times k_\theta)$ vectors of parameters. We will denote by $\varphi = (\sigma_v^2, \sigma_\mu^2, \theta^{'}, (\equiv \{\varphi_p\}^{'} \; p = 1, \ldots, 3)$ the vector of variance-specific parameters.

Staking the T observations of each individual i, (5) may be written as:

$$y_i = x_i\beta + \mu_i \qquad u_i = \iota_T u_i + v_i, \qquad i = 1, \ldots, N \qquad (6)$$

where ι_T is a $(T \times 1)$ vector of ones, and are vectors and a matrix of regressors. From (2), the covariance matrix of may be written as:

$$\Omega_i = \sigma_v^2(I_N \otimes I_T) + \sigma_\mu^2 J_T h_u(f_i^{'}\theta), \qquad i = 1, \ldots, N \qquad (7)$$

where, I_N is an identity matrix of dimension N, I_T is an identity matrix of dimension T, $J_T = \iota_T \iota_T^{'}$ and is a matrix of regressors with a typical row being z_{it}. Here, $diag(h_v(z_i\theta))$ denotes a diagonal matrix with its t^{th} diagonal element being the t^{th} element of the vector.

Finally, stacking again the above vectors and matrices, we obtain the general matrix form of the model.

$$y = X\beta + \mu, \qquad u = Z_\mu \mu + v, \qquad Z_\mu = I_N \otimes \iota_T, \qquad (8)$$

$$\Omega(\theta) = \sigma_v^2 diag(h_v(Z\theta_1)) + \sigma_\mu^2 Z_\mu diag(h_\mu(f\theta))Z_\mu^{'} \qquad (9)$$

y, u and v are $(NT \times 1)$ vectors, μ is the vector of individual effects X, Z and F are, respectively $(NT \times k_\beta)$, $(NT \times K_{\theta 1})$ and $(NT \times k_\theta)$ matrix of regressors and Ω is the $(NT \times NT)$ block-diagonal covariance matrix of u. Here, $diag(h_v(Z\theta)$ denote a diagonal $(NT \times NT)$ matrix with its t^{th} diagonal element being the it^{th} element of the $(NT \times 1)$ vector $(h_v(Z\theta))$.

Data Generating Scheme

The design of our Monte Carlo experiments follows closely that of [1] for panel data, which in turn adapted it from [11]. Consider the following simple regression model:

$$y_{it} = \beta_0 + \beta_1 x_{it} + \mu_i + v_{it} \qquad i = 1, \ldots, N, \qquad t = 1, \ldots, T \qquad (10)$$

where,

$$x_{it} = \omega_{i,t} + 0.5\omega_{i,t-1,} \text{ (see [12])} \qquad (11)$$

We generated $\omega_{i,t}$ as $iid \sim N(0, 2)$ and parameters (β_0, β_1) are assigned (10, 0.5) respectively. We purposedly chose cross-section units as $N = 10, 30, 50,$ and time periods, $T = 5, 15, 20.$ For each scenario, 2500 replication were made. For this work, we set up two experiments:

Experiment 1

Follows the [1] set up, where our contamination was on individual effect, and we assume Quadratic heteroscedastic functional form (QHFF) as stated below: we generated our data by assuming $\mu_i \sim N(0,\sigma^2_{\mu i})$ and our remainder error, $v_{it} \sim N(0,\sigma^2_v)$

$$\sigma^2_{\mu i} = \sigma^2_{\mu i}(\bar{x}_i) = \sigma^2_{\mu}(1+\gamma_{\mu}\bar{x}_i)^2 \quad (12)$$

Experiment 2

We followed the [1] set up, where our contamination was on individual effect, but it took Exponential heteroscedastic functional form (EHFF) as stated below: we generated our data by polluting $\mu_i \sim N(0,\sigma^2_{\mu i})$ and our remainder error, $v_{it} \sim N(0,\sigma^2_v)$

$$\sigma^2_{\mu i} = \sigma^2_{\mu i}(\bar{x}_i) = \sigma^2_{\mu}\exp(\gamma_{\mu}\bar{x}_i) \quad (13)$$

where, $\bar{x}_i$ is the individual mean of x_{it}. Denoting the expected variance of μ_i by $\bar{\sigma}^2_{\mu i}$ and following [1], we fix the expected total variance of $\bar{\sigma}^2_{\mu i} + \sigma^2_v = 10$ to make it comparable across the different data generating processes. We let σ^2_v take the value 4. For each fixed value of σ^2_v, degrees of heteroscedasticity $(DH = \gamma_{\mu})$ is assigned values 0,1,2, and 3 with $\gamma_{\mu} = 0$ denoting the homoscedastic individual specific error. For a fixed value of σ^2_v, we obtained a value of $\bar{\sigma}^2_{\mu i} = (10 - \sigma^2_v)$ and using a specific value of γ_{μ}, we got the corresponding value for σ^2_{μ} from (equations 12 and 13). We can choose a quadratic or an exponential heteroscedastic specification for $\sigma^2_{\mu i} = \sigma^2_{\mu} h_{\mu}(f_i^{'}\theta_2)$ with $h_{\mu}(f_i^{'}\theta_2) = (1+\gamma_{\mu}\bar{x}_i.)^2$ and $h_{\mu}(f_i^{'}\theta_2) = \exp(\gamma_{\mu}\bar{x}_i.)$ for experiments 1 and 2 respectively.

Criteria for Evaluating the Performance of the Estimators

The summary of principal calculations for each model, estimation procedure, hetero-scedasticity function of the individual effect and remainder would be judged with below criteria.

(i) Mean of estimates over replication. Let $\bar{\beta}$, be the estimate of the parameter β obtained in the r^{th} replication, then $\bar{\hat{\beta}} = \frac{1}{R}\sum_{r=1}^{R}\hat{\beta}_r$, where, $R =$ number of replications.

(ii) Bias of the estimator $Bias(\hat{\beta}) = \bar{\hat{\beta}} - \beta$

(iii) Variance $V(\hat{\beta}) = R^{-1}\sum_{r=1}^{R}\left(\hat{\beta}_r - \bar{\hat{\beta}}\right)^2$

(iv) Mean Square Error (MSE)

$$MSE(\beta) = R^{-1}\sum_{r=1}^{R}\left(\hat{\beta}_r - \beta\right)^2$$
$$= Var(\hat{\beta}) + \left[Bias(\hat{\beta})\right]^2$$

(v) Root Mean Square Error (RMSE)

$$RMSE(\hat{\beta}) = \left\{Var(\hat{\beta}) + (Bias(\hat{\beta}))^2\right\}^{\frac{1}{2}}$$

Discussion of Results

Table 1 reports the Monte carlo results of Bias in experiments 1 and 2 when time period T = 5, 15, 20 and cross-section unit N=10 for both experiments. But, it was observed in the Biases of the estimators at $T = 5$ for $N = 10$ and 30, POLS performed reasonably fine but as T and N increases PGLS and BG outperform other estimators, but in Table A3 of appendix, for $N = 50$, PGLS, BG and POLS performed equally and likely in terms of biases.

Table 1: Bias for Slope in Experiments 1 and 2, when N=10 and R=2500

		Experiment 1 , N=10, R=2500				Experiment 2 , N=10, R=2500			
T	γ_μ	**POLS**	**BG**	**WG**	**PGLS**	**POLS**	**BG**	**WG**	**PGLS**
5	0	1.3587	-3.8195	-4.1754	-0.0557	-0.0453	-2.9903	-3.1301	1.4386
	1	-0.6115	1.1080	1.2340	1.1601	3.7151	-0.2130	-0.3544	0.1475
	2	-1.9443	2.9971	3.5171	-1.0562	3.4485	-1.5551	-1.7768	-0.1923
	3	-4.0269	5.6390	6.7138	-2.3847	2.7696	-9.9741	-10.8180	-2.2992
15	0	-0.1374	-0.1290	-0.8928	-0.1087	-0.2667	-0.1562	-0.9772	-0.1539
	1	-0.6044	8.8665	0.6800	-2.3874	2.2656	0.9937	1.0158	0.8864
	2	-4.8641	4.7740	0.7390	-3.1124	8.1738	0.9640	1.0893	0.6936
	3	- 12.9961	-3.1104	0.1086	-4.9578	35.7569	0.8256	1.4326	0.4903
20	0	-0.0454	-1.5228	-0.8538	-0.4471	-0.5869	-1.5228	-0.9264	-0.0536
	1	-0.1042	1.0000	0.8978	0.1664	1.7619	1.0000	1.3641	0.9219
	2	4.2879	1.0000	-0.2665	-4.2742	0.3188	1.0000	0.9356	0.4651
	3	6.3046	1.0000	-1.8477	-10.1225	-4.0761	1.0000	-0.4954	2.7134

[see other tables/figures on appendix page]

Table A4 reports the results of Root Mean Square Estimators (RMSE) for the quadratic (experiment I) form of heteroscedasticity as time period $T = 5$, PGLS and WG recorded minimum RMSE, but in Table 2b as $T = 15$ and 20 $N = 30$, PGLS takes the lead while BG gives the minimum RMSE in Table 3.

Table A5 presents biases on experiment 2 and we found out that PGLS and BG produces minimum biases for all combinations N and T. While the evidences in Table A6 reveal good performance in terms of RMSE criteria to favour PGLS as the best and BG, WG compete as well.

Table 2: Final Summary Results on Both Experiments

Estimator	**Experiment 1**		**Experiment 2**		**Total**	**Performance Ranking**
	BIAS	**RMSE**	**BIAS**	**RMSE**		
POLS	27	16	12	12	67	4TH
BG	23	17	25	15	80	2ND
WG	24	16	20	13	73	3RD
PGLS	31	23	26	20	100	1ST

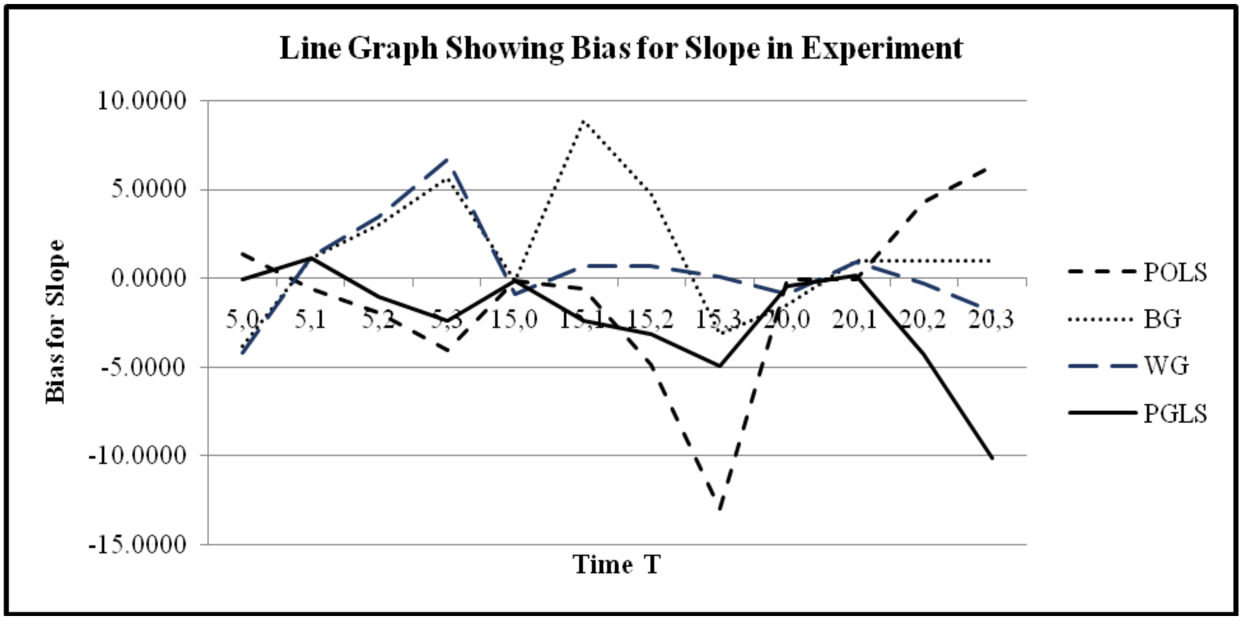

Fig. 1: Graph showing bias performance of different estimators at different time period T around slope.

Conclusion

In this paper, we considered two experiments with four estimators using static panel data regression model. Our Monte carlo and ranking results in experiment 1 shows the following ascending order of performances as; PGLS, POLS, BG and WG. While, for experiment 2, PGLS, BG, WG and POLS. Also, Figure 1 above reveals that as time increases PGLS outperformed better than other estimators.

Generally, considering the two experiments from above Table 2 shows that all estimators were ranked in ascending order of performance: PGLS, BG, WG and POLS. This result helps in the choice of estimator in empirical work when there is presence of heteroscedasticity.

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Appendix

Table A1: Estimated ranking performances on BIAS for Slope coefficient on both experiments

	Experiment 1 , N=10, R=2500						Experiment 2 , N=10, R=2500			
T	γ_μ	**POLS**	**BG**	**WG**	**PGLS**		**POLS**	**BG**	**WG**	**PGLS**
5	0	1.3587	-3.8195	-4.1754	-0.0557		-0.0453	-2.9903	-3.1301	1.4386
	1	-0.6115	1.1080	1.2340	1.1601		3.7151	-0.2130	-0.3544	0.1475
	2	-1.9443	2.9971	3.5171	-1.0562		3.4485	-1.5551	-1.7768	-0.1923
	3	-4.0269	5.6390	6.7138	-2.3847		2.7696	-9.9741	-10.8180	-2.2992
15	0	-0.1374	-0.1290	-0.8928	-0.1087		-0.2667	-0.1562	-0.9772	-0.1539
	1	-0.6044	8.8665	0.6800	-2.3874		2.2656	0.9937	1.0158	0.8864
	2	-4.8641	4.7740	0.7390	-3.1124		8.1738	0.9640	1.0893	0.6936
	3	-12.9961	-3.1104	0.1086	-4.9578		35.7569	0.8256	1.4326	0.4903
20	0	-0.0454	-1.5228	-0.8538	-0.4471		-0.5869	-1.5228	-0.9264	-0.0536
	1	-0.1042	1.0000	0.8978	0.1664		1.7619	1.0000	1.3641	0.9219
	2	4.2879	1.0000	-0.2665	-4.2742		0.3188	1.0000	0.9356	0.4651
	3	6.3046	1.0000	-1.8477	-10.1225		-4.0761	1.0000	-0.4954	2.7134

Table A2: Estimated Ranking Performances on BIAS for Slope Coefficient on both Experiments

	Experiment 1, N=30, R=2500					Experiment 2, N=30, R=2500			
T	γ_μ	**POLS**	**BG**	**WG**	**PGLS**	**POLS**	**BG**	**WG**	**PGLS**
5	0	-0.4205	-0.5010	0.0402	-0.1711	-0.4987	-0.7361	-0.1895	-0.1917
	1	0.2293	0.5743	17.5124	0.5932	1.1374	1.9766	3.0913	1.2817
	2	-1.2128	-0.2220	48.4005	0.3402	1.7790	6.5361	12.8528	0.8957
	3	-3.3256	-1.3888	93.6617	0.2665	4.7750	27.8223	58.4253	0.6406
15	0	-0.7371	-0.9431	0.1015	0.0064	-0.4636	-0.2705	-0.3642	0.0089
	1	0.9722	0.4896	-5.8404	-1.4688	1.1734	-0.7778	-0.3784	-6.2135
	2	1.5570	0.5789	-7.1547	-1.3906	1.6221	-7.3323	-8.3704	-12.9551
	3	2.4028	0.6952	-3.5350	-18.5313	5.4715	-44.8344	-36.9228	3.8989
20	0	-0.4223	0.1163	-0.3123	-0.4873	-0.3976	-0.6965	-0.2869	-0.0759
	1	0.8603	-0.4961	0.2834	-0.0408	1.0735	0.7248	-0.3591	1.6665
	2	-0.4433	2.3758	-1.7323	-0.8122	2.4165	2.9411	4.2125	2.4299
	3	-1.5179	3.3534	-2.5034	-1.6021	8.0212	10.9233	14.7590	8.7575

Table A3: Estimated Ranking Performances on BIAS for Slope Coefficient on both Experiments

	Experiment 1, N=50, R=2500					Experiment 2, N=50, R=2500			
T	γ_μ	**POLS**	**BG**	**WG**	**PGLS**	**POLS**	**BG**	**WG**	**PGLS**
5	0	-0.2200	-0.1679	0.1701	0.0367	-0.1791	-0.1916	-0.3054	-0.1892
	1	-0.4586	-0.3734	3.3292	1.4005	1.7409	0.1221	1.7040	1.6607
	2	-0.3609	-0.1394	8.3163	0.0163	4.0614	0.5376	0.0026	-2.4857
	3	-7.1864	-4.6468	7.7437	-21.6948	14.3400	-0.4113	-0.4741	-21.1126
15	0	-0.5576	-0.8658	-0.5633	-0.0345	-0.5513	-1.0160	-0.0573	-0.5801
	1	1.1860	0.1684	0.4186	-0.2551	0.9380	0.7057	1.6265	0.5732
	2	0.6067	-0.9584	-1.9707	2.6290	1.0941	-0.5924	5.7592	-0.7487
	3	0.2328	-2.4172	-3.0581	8.1478	1.2750	-4.5104	9.0576	-8.0946
20	0	-0.4783	-0.2002	-0.0857	0.0307	-0.4917	-0.0350	0.0095	0.1042
	1	1.0383	1.1249	5.4259	-0.8730	1.0893	1.3714	11.8421	0.1363
	2	0.8741	3.2213	2.4492	-3.9121	1.3285	1.6800	73.8737	7.0115
	3	1.0157	5.1169	13.5963	-5.0326	2.4749	5.4829	387.5474	34.5518

Table A4: Estimated Ranking Performances on RMSE for Slope Coefficient on both Experiments

		Experiment 1, N=10, R=2500				Experiment 2, N=10, R=2500			
T	γ_μ	**POLS**	**BG**	**WG**	**PGLS**	**POLS**	**BG**	**WG**	**PGLS**
5	0	1.3693	3.8297	4.1803	0.2720	1.4486	3.0034	3.1367	0.2652
	1	0.6194	1.1075	1.2335	0.9515	0.1876	0.2317	0.3595	1.1702
	2	1.9332	2.9900	3.5133	0.9623	0.2241	1.5511	1.7742	1.2831
	3	3.9977	5.6244	6.7060	1.1384	2.2864	9.9310	10.7955	2.4117
15	0	0.1663	0.4452	0.8972	0.1102	0.2827	0.4539	0.9822	0.1872
	1	0.5419	2.5890	0.6771	2.3691	2.1631	0.9993	1.0156	0.9279
	2	4.0380	1.4127	0.7375	3.1087	7.7191	0.9958	1.0884	0.8128
	3	10.7534	0.9583	0.1417	5.2782	33.7383	0.9810	1.4289	0.7035
20	0	0.4555	1.5943	0.8583	0.1098	0.5933	1.5943	0.9306	0.1126
	1	0.2513	1.0000	0.8989	0.8957	1.7490	1.0000	1.3613	0.9498
	2	4.1999	1.0000	0.2843	2.3226	0.3478	1.0000	0.9362	0.9026
	3	9.9380	1.0000	1.8408	4.3970	4.0347	1.0000	0.5022	2.0267

Table A5: Estimated Ranking Performances on RMSE for Slope Coefficient on both Experiments

		Experiment 1, N=30, R=2500				Experiment 2, N=30, R=2500			
T	γ_μ	**POLS**	**BG**	**WG**	**PGLS**	**POLS**	**BG**	**WG**	**PGLS**
5	0	0.4316	0.5433	0.1183	0.2064	0.5080	0.7655	0.2198	0.2233
	1	0.3172	0.6558	6.0254	0.7587	1.1329	1.9204	2.7125	1.2297
	2	1.2027	0.4377	16.4742	0.6555	1.7569	6.2911	11.0924	1.0031
	3	3.2479	1.3379	31.8411	0.6342	4.6915	26.7552	50.3735	0.8934
15	0	0.7386	0.0315	0.1026	0.9455	0.4670	0.3502	0.3689	0.0714
	1	0.9732	2.2952	5.8070	0.5420	1.1711	0.8744	0.4066	1.1261
	2	1.5558	2.3016	7.1014	0.6236	1.6188	5.6996	8.2651	1.8880
	3	2.3989	3.8444	3.5000	0.7043	5.4330	34.6368	36.4717	1.4412
20	0	0.3161	0.5268	0.4250	0.1283	0.4004	0.7247	0.1037	0.2910
	1	0.3309	0.3818	0.8620	0.4538	1.0880	0.7104	1.2672	0.3679
	2	1.7178	0.8214	0.4673	2.1621	2.4136	3.1639	2.0263	4.2110
	3	2.4915	1.5638	1.5364	3.0607	8.0692	12.1585	6.7489	14.7046

Table A6: Estimated Ranking Performances on RMSE for Slope Coefficient on both Experiments

		Experiment 1, N=50, R=2500				Experiment 2, N=50, R=2500			
T	γ_μ	**POLS**	**BG**	**WG**	**PGLS**	**POLS**	**BG**	**WG**	**PGLS**
5	0	0.2329	0.2395	0.1879	0.0914	0.2040	0.1809	0.3086	0.2487
	1	0.5432	0.7595	3.0437	1.1457	1.6550	1.8016	1.6944	0.1393
	2	0.6777	0.1821	7.5419	1.0233	2.4300	4.0327	0.3954	0.4938
	3	6.7966	3.2814	7.1185	8.8249	19.8851	14.2689	1.7003	1.9403
15	0	0.0460	0.8782	0.5651	0.5592	0.5529	1.0265	0.0758	0.5818
	1	1.3413	0.1737	0.4187	1.1858	0.9401	0.7004	1.2454	0.5720
	2	3.2783	0.9596	1.9660	0.6096	1.0950	0.6218	4.3918	0.7549
	3	6.1500	2.4620	3.0541	0.2441	1.6477	4.8208	6.9235	8.1157
20	0	0.4796	0.2483	0.0899	0.0332	0.4930	0.1604	0.0100	0.0353
	1	1.0363	0.9166	5.1735	0.8825	1.0868	0.1440	11.2700	1.3626
	2	3.6355	2.6097	2.3471	0.8739	1.3284	4.5567	70.1900	1.6827
	3	1.0448	4.1659	12.9722	4.8886	2.4779	22.4601	368.2100	5.6204

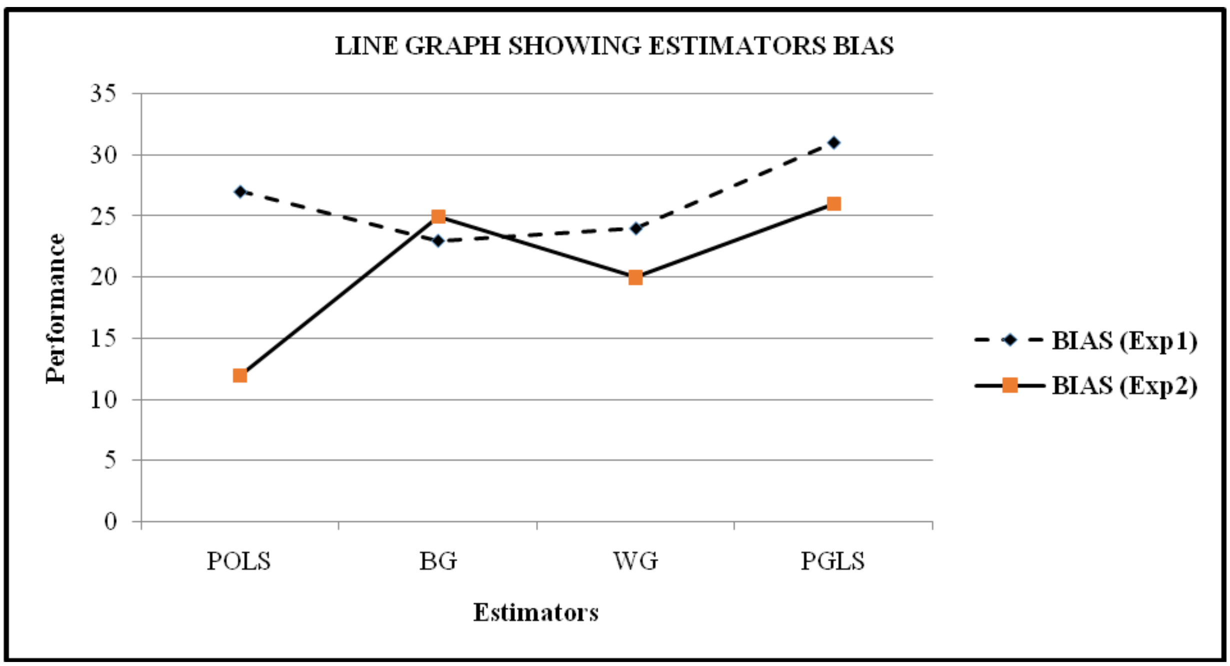

Fig. 2: The graph showing bias performance of different estimators on both experiments.

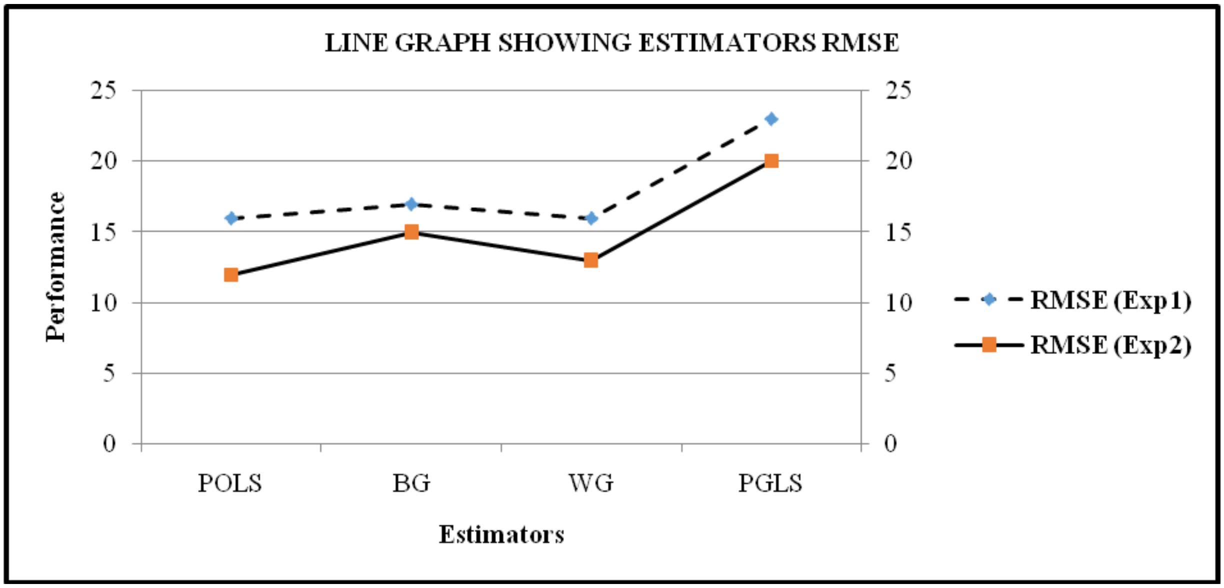

Fig. 3: The graph showing RMSE performance of different estimators on both experiments.

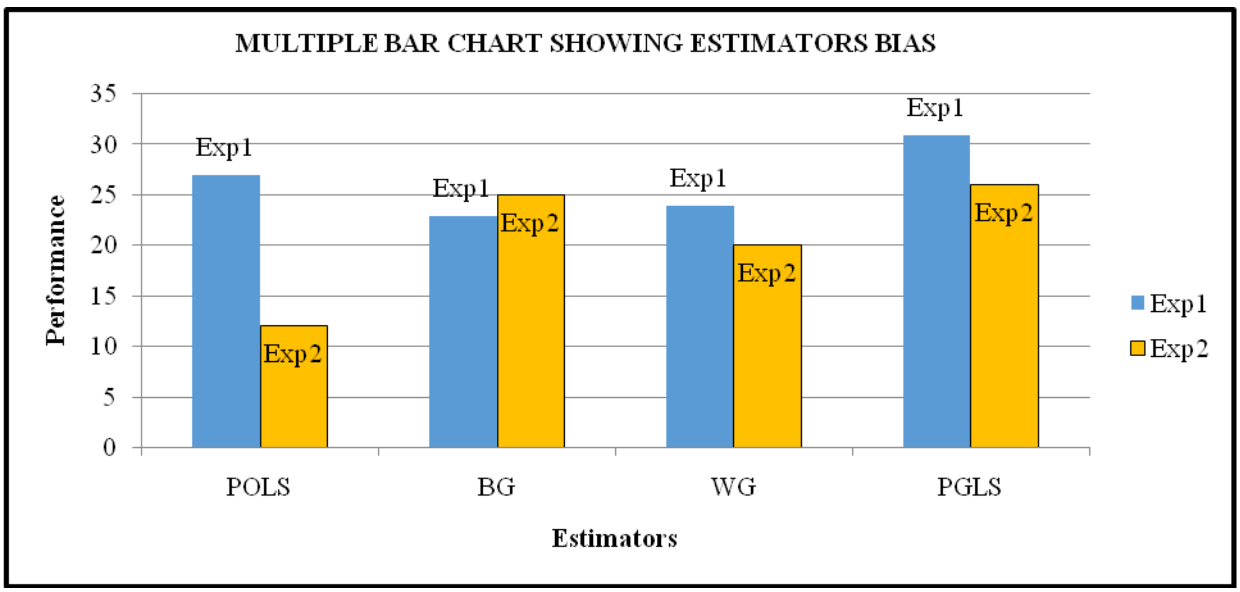

Fig. 4: The multiple bar chart showing bias performance of different estimators on both experiments.

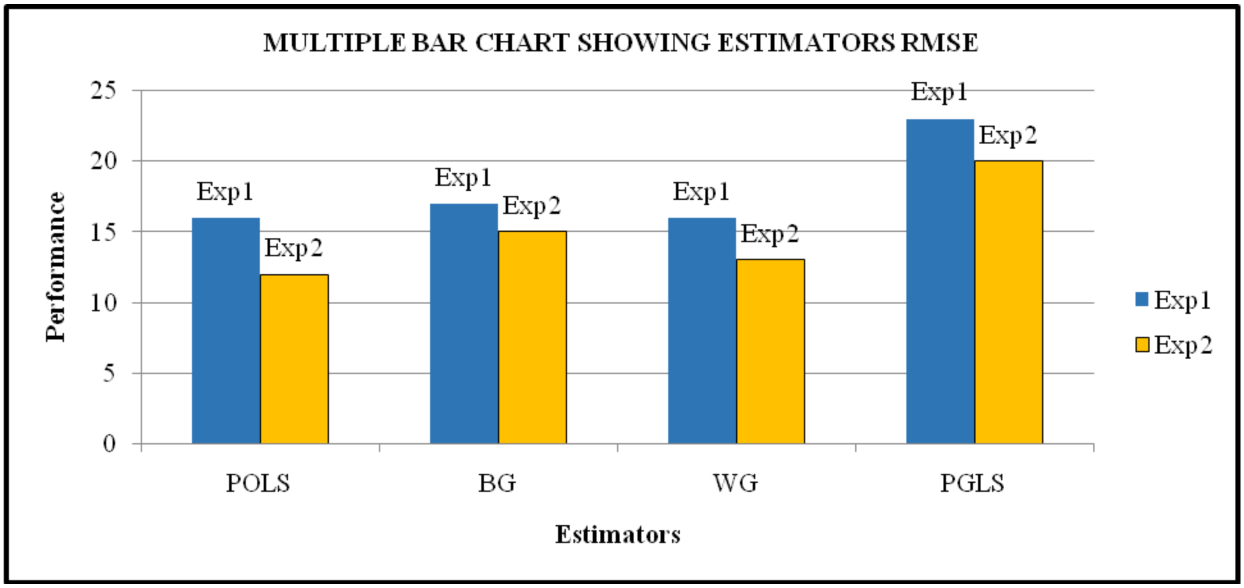

Fig. 5: The multiple bar chart showing RMSE performance of different estimators on both experiments.