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A Tableau Proof System for the Spatial Qualification Logic

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Abstract

Spatial Qualification Model (SQM) is a logical system built to reason about the possibility of an agent's presence at a location of incidence, at a certain time given its prior antecedents. The usability of this logical system is strongly dependent on the existence of a proof system. This work is aimed at developing an analytic proof system to demonstrate the logical truth of statements in the spatial qualification logic. Tableau proof method was used to analytically prove that the formulae (axioms) in the SQM system were the logical consequence of the set of formulae that make up the prior knowledge. The proof system confirms the logical truth of the axioms in the system.

Key words: Spatial qualification model, Tableau proof method, Logical proof system.

Introduction

The logical model of spatial qualification [1] was formalized following the introduction of the spatial qualification problem [2, 3]. The problem is a specific type of qualification problem that deals with the impossibility of knowing an intelligent agent's presence at a specific location and time to carry out an action or participate in an event given its known antecedents. Spatial qualification therefore, is one important precondition for any action to take place at a location. The qualitative spatial reasoning field have formalized a number of spatial concepts. Some of the resulting qualitative spatio-temporal calculi used for reasoning about these concepts are included in the summary in Table 1.

The spatial qualification model [1] is a logical system with axioms directed towards the determination of the spatial presence of an agent for it to be a participant in an action with the reachability of the locations concerned taken into consideration. The axioms contained in the logic re-used the RCC-8 topological relations [6] in defining spatial concepts, such as regionally_ disjoint and regionally_ connected [1], required to determine the possibility of an agent's spatial qualification. Formal axioms that make up the spatial qualification model (SQM) are explicitly stated following the quantified modal logic [11]. The formal model was described using the possible world semantics [12] with the concepts of the world remaining fixed across possible worlds confirming Barcan's axioms [13] with constant domain across possible worlds.

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Table 1: Review of Qualitative Spatiotemporal Calculi

Calculi	**Relations**	**Examples**
Star, double cross, cardinal direction [4]	direction	left, above,…
Point, relative distance	distance	far, near,…
Sizes	sizes	large, tiny,…
Shapes [5]	shapes	oval, convex,…
RCC-5, RCC-8 [6]	topology	touch, inside,…
QTC [7,8]	motion	reach, leave, hit…
Allen Interval logic [9]	Interval Relations	before, after, …
Anchoring relations [10]	Anchoring relations	around, within
SQM - Spatial Qualification Model [1]	Spatial Qualification	present at, reachable

The validity of the logical statements in spatial qualification model (SQM) can only be determined if there is a mechanical procedure for determining whether or not the statement follows from a given set of statements. Theorem proofing can be done using several proof methods such as unification, resolution and tableau proof method [14]. Classical logics such as first order logic can best be handled using resolution since it is already in clausal form. The most widely used proof method for modal logics is the analytic/semantic tableau proof method. The use of analytic tableau proof method reduces the burden of transforming sentences such as possibility to its clausal form. Semantic tableau is a proof system used to prove the validity of a formula, or if a formula is a logical consequence of a set of formulas and/or prove of satisfiability of a set of formulae [15]. Since, the logic follows the syntax of the quantified modal logic, tableau proof method is said to be appropriate for the proof system. This work is aimed at employing the analytical tableau proof method to provide a proof system for the logical model of spatial qualification.

The rest of the paper is organized as follows. Section 2 gives the review of Quantified Modal Logics. An overview of the methodology used and the proof system is described in section 3. Section 4 discusses the decidability of the proof system. Section 5 gives the summary and conclusion of the paper.

Quantified Modal Logics

Quantified modal logic combines features of two logical languages, namely—the adequate expressivity of first-order logic and the dynamic modalities of the modal logic [16]. Clearly outlined is the syntax of first-order logic [17] and its semantics [18]. The syntax and semantics of modal logic is as outlined in [19, 20].Hence, the operators used in quantified modal logics include the universal quantifier $\forall$, existential quantifier $\exists$, unary operator $\neg$, binary operators $\wedge$, $\vee$,$\Rightarrow$, $\Leftrightarrow$, the necessarily $\Box$ and the possibly operator $\Diamond$. The complete semantics of quantified modal logic has been developed [21].

Although, semantic tableau has been used to prove the validity of formulae in propositional and predicate logic [15], it is more widely used as a proof technique for modal systems. It has also been used in providing a sound and complete proof system for the propositional S5 system [19]. Fitting [13] in his attempt to prove the validity of Barcan's formula and its converse, which is of constant domain across possible worlds, gave several examples of propositions and their tableau proofs. This can also be seen in the tableau proof whose rules go with the semantics of the first-order modal logic [11].

Semantic tableau is the most suitable proof method for modal logic [14]. Hence, theorem proving of the spatial qualification logic which employs the quantified modal logic for its representation adopts the tableau proof method to ascertain its logical truth.

The Tableau Proof System

The most widely used proof method for modal logics is the analytic tableau proof method. Semantic tableau is a proof system used to prove the validity of a formula, or if a formula is a logical consequence of a set of formulas and/or prove of validity of a set of formulae [14]. A tableau is a tree-like representation of a formula or set of formulae in logic [22]. Each node of a tableau carries a signed formula (true or false). Tableau calculus consists of a finite collection of rules. Rules specify how to break down one logical connective into its constituent parts.

A proof procedure on the other hand is a policy for application of the rules. The application of a tableau rule following a finite path sometimes leads to the creation of many branches. In tableaux, if any branch leads to an evident contradiction, the branch closes. If all branches close, the proof is complete and the original formula is said to be a logical truth. The objective of tableaux is to show that the negation of a formula is not valid.

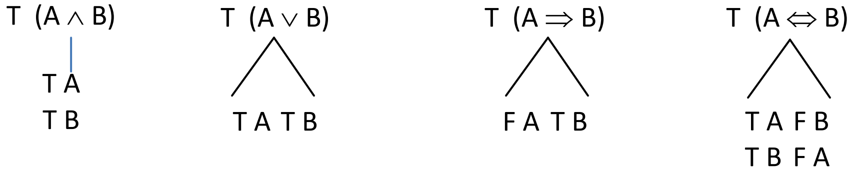
The basic rules for constructing the tableau stems from that of propositional logic, extends to rules that deal with the universal and existential quantifiers in first-order logic and then to rules that deal with *possibly* and *necessarily* modalities of modal logics. Since our logic is a quantified modal logic, we shall combine all the rules for propositional, first-order and modal logics as described in the section addressing the development of the proof system. First-order logic is said to be undecidable in general. This means that no procedure exists in general that can tell whether or not a statement is valid. However, it is semi-decidable in the sense that a procedure exists that can tell if a statement is valid, but the procedure is not guaranteed to terminate if the statement is not valid. We look forward to seeing the case of modal logics.

Tableau Proof Rules

Since, SQM follows the syntax of quantified modal logic which combines features of first-order and that of modal logic, our proof rules therefore will combine the tableau rules in propositional [22], first-order and modal logic. Hence, the following tableau rules shall become applicable to the proof system of SQM.

(i) Negation rules

(ii) Conjunctive rules



(iii) Disjunctive rules

$$\begin{array}{c} F\ (A \wedge B) \\ \diagup \quad \diagdown \\ F\,A \quad F\,B \end{array} \qquad \begin{array}{c} F\ (A \vee B) \\ | \\ T\,A \\ T\,B \end{array} \qquad \begin{array}{c} F(A \Rightarrow B) \\ | \\ T\,A \\ F\,B \end{array} \qquad \begin{array}{c} F\ (A \Leftrightarrow B) \\ \diagup \quad \diagdown \\ T\,A \quad F\,A \\ F\,B \quad T\,B \end{array}$$

(iv) Universal rules

$$\begin{array}{c} T\ \forall x\ \ A(x) \\ | \\ T\ A(t) \end{array} \qquad \begin{array}{c} F\ \exists x\ \ A(x) \\ | \\ F\ A(t) \end{array}$$

for any term t in the language.

(v) Existential rules

$$\begin{array}{c} T\ \exists x\ \ A(x) \\ | \\ T\ A(c) \end{array} \qquad \begin{array}{c} F\ \forall x\ \ A(x) \\ | \\ F\ A(c) \end{array}$$

for a new constant c.

(vi) Necessity rules

$$\begin{array}{c} T\ \Box A \\ | \\ T_k\ A \end{array} \qquad \begin{array}{c} F\ \Diamond A \\ | \\ F_k\ A \end{array}$$

(vii) Possibility rules

$$\begin{array}{c} T\ \Diamond A \\ | \\ T_k\ A \end{array} \qquad \begin{array}{c} F\ \Box A \\ | \\ F_k\ A \end{array}$$

Sample Tableau proofs for SQM

To prove that a formula, B is a logical consequence of a set of formulae, A_1 ... A_k, the following lemmas are hereby stated.

Lemma 1. Given that:

$\{Present\_at(x,l_1,t_1),\ \ \forall x,l_1,l_2,t_1,t_2.\ \ l_1=l_2 \wedge t_1<t_2 \Rightarrow Reachable(x,l_1,l_2,(t_1,t_2)),\ \ l_1=l_2,\ t_1<t_2\}$
$\nvdash\ \Diamond Present\_at(x,l_2,t_2)$

Proof: To proof by contradiction that the above lemma is true, we start by saying that the set of axioms entails $\neg\Diamond Present\_at(x,l_2,t_2)$. Including the negated axiom to the set and proving using tableau rules is as analysed in Figure 1 and completed in Figure 2. Since both branches of the above tableau do not lead to a closure, we look for a way of extending the branch that is still open.

From the system of axioms in SQM, axiom T_{A3} defines *Reachable* to be $Reachable(x,l_1,l_2,(t_1,t_2)) \Leftrightarrow (t_1<t_2 \wedge (Presen\_at(x,l_1,t_1) \Rightarrow \Diamond Present\_at(x,l_2,t_2)).$

By equivalence, we have that

$Reachable(x,l_1,l_2,(t_1,t_2)) = (t_1<t_2 \wedge (Present\_at(x,l_1,t_1) \Rightarrow \Diamond Present\_at(x,l_2,t_2))$.

By substitution rule, $(t_1<t_2 \wedge (Present\_at(x,l_1,t_1) \Rightarrow \Diamond Present\_at(x,l_2,t_2))$ replaces $Reachable(x,l_1,l_2,(t_1,t_2))$ in the tableau in figure 1 and thus extends the branch further in other to lead to closure as shown in tableau 2.

1. $\{Present\_at(x,l_1,t_1),\ \forall x,l,t_1,t_2.\ .\ l_1=l_2 \wedge t_1<t_2 \Rightarrow Reachable(x,l_1,l_2,(t_1,t_2)),\ t_1<t_2,\ l_1=l_2, \neg\Diamond Present\_at(x,l_2,t_2)\}$
2. $Present\_at(x,l_1,t_1)$
3. $\forall x,l_1,l_2,t_1,t_2.\ l_1=l_2 \wedge t_1<t_2 \Rightarrow Reachable(x,l_1,l_2,(t_1,t_2))$
4. $t_1< t_2$
5. $l_1 = l_2$
6. $\neg\Diamond Present\_at(x,l_2,t_2)$
7. $\neg Present\_at(x,l_2,t_2)$
8. $l_1=l_2 \wedge t_1<t_2 \Rightarrow Reachable(x,l_1,l_2,(t_1,t_2))$
9. $\neg (l_1=l_2 \wedge t_1<t_2)$
10. $Reachable(x,l_1,l_2,(t_1,t_2))$
11. $\neg (l_1=l_2)$
12. $\neg (t_1<t_2)$

Fig. 1: Proof of reachability (reflexive) axiom – open.

1. $\{Present\_at(x,l_1,t_1),\ \forall x,l,t_1,t_2.\ .\ l_1=l_2 \wedge t_1<t_2 \Rightarrow Reachable(x,l_1,l_2,(t_1,t_2)),\ t_1<t_2,\ l_1=l_2,\neg\Diamond Present\_at(x,l_2,t_2)\}$
2. $Present\_at(x,l_1,t_1)$
3. $\forall x,l_1,l_2,t_1,t_2.\ l_1=l_2 \wedge t_1<t_2 \Rightarrow Reachable(x,l_1,l_2,(t_1,t_2))$
4. $t_1< t_2$
5. $l_1 = l_2$
6. $\neg\Diamond Present\_at(x,l_2,t_2)$
7. $\neg Present\_at(x,l_2,t_2)$
8. $l_1=l_2 \wedge t_1<t_2\Rightarrow Reachable(x,l_1,l_2,(t_1,t_2))$
9. $l_1=l_2 \wedge t_1<t_2$
10. $Reachable(x,l,l,(t_1,t_2))= (t_1<t_2\wedge(Present\_at(x,l_1,t_1)\Rightarrow\Diamond Present\_at(x,l_2,t_2))$
11. $\neg\ (l_1=l_2)$
12. $\neg\ (t_1<t_2)$
13. $t_1<t_2\wedge(Present\_at(x,l_1,t_1)\Rightarrow\Diamond Present\_at(x,l_2,t_2))$
14. $t_1< t_2$
15. $(Present\_at(x,l_1,t_1) \Rightarrow \Diamond Present\_at(x,l_2,t_2))$
16. $\neg Present\_at(x,l_1,t_1)$
17. $\Diamond Present\_at(x,l_2,t_2)$
18. $Present\_at(x,l_2,t_2)$

Fig. 2: Proof of reachability (reflexive) axiom - closed.

From the closed tableau in Figure 2, the set of axioms in node *(1)* is expanded to have nodes *(2)*, *(3)*, *(4)*, *(5)* and *(6)*. By applying necessity rule from *(6)*, we have item *(7)*. Node *(8)* is from node *(3)* by universal rule. By conjunctive rule, node *(8)* opens into two branches with nodes *(9)* and *(10)*. Node *(9)* again opens into two branches with nodes *(11)* and *(12)* and nodes *(11)* and *(5)* close as well as nodes *(12)* and *(4)* since there is a contradiction.

Extending node *(10)* as explained in the first tableau above, we have node *(10)* replaced as shown in the second tableau in Figure 2. By substitution rule, we have node *(13)* from *(10)*. Again by conjunctive rule from node *(13)*, we have nodes *(14)* and *(15)*. Node *(15)* opens into two branches with nodes *(16)* and *(17)* by conjunctive rule. By possibility rule from *(17)*, we have node *(18)*. All branches in the tableau in Figure 2 lead to closure as nodes *(17)* and *(2)* close and also nodes *(18)* and *(7)* close.

The closure of the tableau of the contradiction shows that the proof is complete and that SQM logic is satisfiable with the given statement in lemma 1.

Lemma 2. Given that:

$\{Present\_at(x,l_1,t_1),$
$Reachable(x,l_1,l_2,(t_1,t_2)),$
$Reachable(x,l_2,l_3,(t_2,t_3)), t_1<t_2<t_3,$
$Reachable(x,l_1,l_2,(t_1,t_2)) \wedge Reachable(x,l_2,$
$l_3,(t_2,t_3)) \Rightarrow Reachable(x,l_1,l_3,(t_1,t_3))\}$
$\vdash \Diamond Present\_at(x,l_3,t_3)$

Proof: The proof is as shown on Figure 3 and completed in Figure 4.

On applying the branch extension rule as it was done in figure 2, we have from axiom T_{A3}, the tableau as shown in Figure 4.

1. $\{Present\_at(x,l_1,t_1), Reachable(x,l_1,l_2,(t_1,t_2)), Reachable(x,l_2,l_3,(t_2,t_3)), t_1<t_2<t_3, Reachable(x,l_1,l_2,(t_1,t_2)) \wedge Reachable(x,l_2,l_3,(t_2,t_3)) \Rightarrow Reachable(x,l_1,l_3,(t_1,t_3)), \neg \Diamond Present\_at(x,l_3,t_3)\}$
2. $Present\_at(x,l_1,t_1)$
3. $Reachable(x,l_1,l_2,(t_1,t_2))$
4. $Reachable(x,l_2,l_3,(t_2,t_3))$
5. $t_1 < t_2 < t_3$
6. $\forall x,l_1,l_2,l_3,t_1,t_2,t_3.\ Reachable(x,l_1,l_2,(t_1,t_2)) \wedge Reachable(x,l_2,l_3,(t_2,t_3)) \Rightarrow Reachable(x,l_1,l_3,(t_1,t_3))$
7. $\neg \Diamond Present\_at(x,l_3,t_3)$
8. $\neg Present\_at(x,l_3,t_3)$
9. $Reachable(x,l_1,l_2,(t_1,t_2)) \wedge Reachable(x,l_2,l_3,(t_2,t_3)) \Rightarrow Reachable(x,l_1,l_3,(t_1,t_3))$
10. $\neg (Reachable(x,l_1,l_2,(t_1,t_2)) \wedge Reachable(x,l_2,l_3,(t_2,t_3))$
11. $Reachable(x,l_1,l_3,(t_1,t_3))$
12. $\neg Reachable(x,l_1,l_2,(t_1,t_2))$
13. $\neg Reachable(x,l_2,l_3,(t_2,t_3))$

Fig. 3: Proof of reachability (transitive) axiom- open.

1. $\{Present\_at(x,l_1,t_1),\quad Reachable(x,l_1,l_2,(t_1,t_2)),Reachable(x,l_2,l_3,(t_2,t_3)), t_1<t_2<t_3,Reachable(x,l_1,l_2,(t_1,t_2))\wedge Reachable(x,l_2,l_3,(t_2,t_3))\Rightarrow Reachable(x,l_1,l_3,(t_1,t_3)),\neg\Diamond Present\_at(x,l_3,t_3)\}$
2. $Present\_at(x,l_1,t_1)$
3. $Reachable(x,l_1,l_2,(t_1,t_2))$
4. $Reachable(x,l_2,l_3,(t_2,t_3))$
5. $t_1< t_2 < t_3$
6. $\forall x,l_1,l_2,l_3,t_1,t_2,t_3.\quad Reachable(x,l_1,l_2,(t_1,t_2))\wedge Reachable(x,l_2,l_3,(t_2,t_3))\Rightarrow Reachable(x,l_1,l_3,(t_1,t_3))$
7. $\neg\Diamond Present\_at(x,l_3,t_3)$
8. $\neg Present\_at(x,l_3,t_3)$
9. $Reachable(x,l_1,l_2,(t_1,t_2))\wedge Reachable(x,l_2,l_3,(t_2,t_3))\Rightarrow Reachable(x,l_1,l_3,(t_1,t_3))$
10. $\neg\ (Reachable(x,l_1,l_2,(t_1,t_2))\wedge Reachable(x,l_2,l_3,(t_2,t_3))$
11. $Reachable(x,l_1,l_3,(t_1,t_3))= t_1<t_3\wedge Present\_at(x.l_1,t_1) \Rightarrow\Diamond Present\_at(x,l_3,t_3)$
12. $\neg Reachable(x,l_1,l_2,(t_1,t_2))$
13. $\neg Reachable(x,l_2,l_3,(t_2,t_3))$
14. $(t_1<t_3\wedge(Present\_at(x,l_1,t_1)\Rightarrow\Diamond Present\_at(x,l_3,t_3))$
15. $t_1< t_3$
16. $(Present\_at(x,l_1,t_1) \Rightarrow \Diamond Present\_at(x,l_3,t_3))$
17. $\neg Present\_at(x,l_1,t_1)$
18. $\Diamond Present\_at(x,l_3,t_3)$
19. $Present\_at(x,l_3,t_3)$

Fig. 4: Proof of reachability (transitive) axiom -closed.

Lemma 3. Given that:

$\{Present\_at(x,l_1,t_1), \forall x,l_1,l_2,t_1,t_2.\ l_1=l_2 \wedge t_1<t_2 \Rightarrow Reachable(x,l_1,l_2,(t_1,t_2)), t_1<t_2\}$
$\nvdash \Diamond Present\_at(x,l_2,t_2)$

Proof: To prove by contradiction that the above lemma is true, we start by saying that the set of axioms entails $\neg(\neg\Diamond Present\_at(x,l_2,t_2))$. Including to the set and proofing using tableau rules is as shown in Figure 5. and completed in Figure 6. By employing substitution rule as seen in the proof of lemma 1 and replacing, $Reachable(x,l_1,l_2,(t_1,t_2))$ of node *(11)* in Figure 5 with $(l_1=l_2 \wedge t_1<t_2 \wedge (Present\_at(x,l_1,t_1) \Rightarrow \Diamond Present\_at(x,l_2,t_2))$, and thus extends the branch further in other to lead to closure as shown in Figure 6.

1. $\{Present\_at(x,l_1,t_1),\ \forall x,l_1,l_2,t_1,t_2.\ l_1=l_2 \wedge t_1<t_2 \Rightarrow Reachable(x,l_1,l_2,(t_1,t_2)),\ t_1<t_2,$ $l_1=l_2, \neg(\neg\Diamond Present\_at(x,l_2,t_2))\}$
2. $Present\_at(x,l_1,t_1)$
3. $\forall x,l,t_1,t_2.\ l_1=l_2 \wedge t_1<t_2 \Rightarrow Reachable(x, l_1,l_2,(t_1,t_2))$
4. $t_1<t_2$
5. $l_1=l_2$
6. $\neg(\neg\Diamond Present\_at(x,l_2,t_2))$
7. $\Diamond Present\_at(x,l_2,t_2)$
8. $Present\_at(x,l_2,t_2)$
9. $l_1=l_2 \wedge t_1<t_2 \Rightarrow Reachable(x, l_1,l_2,(t_1,t_2))$
10. $\neg\ (l_1=l_2 \wedge t_1<t_2)$
11. $Reachable(x,l_1,l_2,(t_1,t_2))$
12. $\neg\ (l_1=l_2)$
13. $\neg\ (t_1<t_2)$

Fig. 5: Proof of negation of reachability (reflexive) axiom - open.

1. $\{Present\_at(x,l_1,t_1), \forall x,l,t_1,t_2. . l_1=l_2 \wedge t_1<t_2 \Rightarrow Reachable(x,l_1,l_2,(t_1,t_2)), t_1<t_2, l_1=l_2, ,\neg(\neg\Diamond Present\_at(x,l_2,t_2))\}$
2. $Present\_at(x,l_1,t_1)$
3. $\forall x,l_1,l_2,t_1,t_2. l_1=l_2 \wedge t_1<t_2 \Rightarrow Reachable(x,l_1,l_2,(t_1,t_2))$
4. $t_1< t_2$
5. $l_1 = l_2$
6. $\neg(\neg\Diamond Present\_at(x,l_2,t_2))$
7. $\Diamond Present\_at(x,l_2,t_2)$
8. $Present\_at(x,l_2,t_2)$
9. $l_1=l_2 \wedge t_1<t_2 \Rightarrow Reachable(x,l_1,l_2,(t_1,t_2))$
10. $\neg (l_1=l_2 \wedge t_1<t_2)$
11. $Reachable(x,l,l,(t_1,t_2))=$
    $(l_1=l_2 \wedge t_1<t_2 \wedge (Present\_at(x,l_1,t_1) \Rightarrow \Diamond Present\_at(x,l_2,t_2))$
12. $\neg (l_1=l_2)$
13. $\neg (t_1<t_2)$
14. $(t_1<t_2 \wedge (Present\_at(x,l_1,t_1) \Rightarrow \Diamond Present\_at(x,l_2,t_2))$
15. $t_1< t_2$
16. $(Present\_at(x,l_1,t_1) \Rightarrow \Diamond Present\_at(x,l_2,t_2))$
17. $\neg Present\_at(x,l_1,t_1)$
18. $\Diamond Present\_at(x,l_2,t_2)$
19. $Present\_at(x,l_2,t_2)$

Fig. 6: Proof of negation of reachability (reflexive) axiom - closed.

From the tableau in Figure 6, the set of axioms in node *(1)* is expanded to have nodes *(2), (3), (4), (5)* and *(6).* Node *(7)* is obtained by applying double negation rule on *(6).* And by necessity rule from *(7),* we have item *(8).* Node *(9)* is from node *(3)* by universal rule. By conjunctive rule, node *(9)* opens into two branches with nodes *(10)* and *(11)* and by disjunctive rule node *(10)* extends to nodes *(12)* and *(13).* Nodes *(12)* and node *(5)* close and nodes *(13)* and *(4)* also close.

As seen in Figure 2, we have node *(11)* replaced as shown in the second tableau. By substitution rule, we have node *(14)* from *(11).* Again by conjunctive rule from node *(14),* we have nodes *(15)* and *(16).* Node *(16*) opens into two branches with nodes *(17)* and *(18)* by conjunctive rule. By possibility rule from *(18)*, we have node *(19)*. Nodes *(17)* and *(2)* close but node *(19)* is open.

Since the branches in the tableau of Figure 6 do not all lead to closure, the proof is therefore incomplete showing that the negation of the assertion is not provable. This demonstrates the semi-decidability of SQM. Note the proofs, started by including the contradiction of the entailed formula to the right hand side of the statements to the set. Since, the tableau proofs with the contradiction lead to closure, it means that, the starting set of formulae is not a logical truth. This means that the set with the originally entailed formula is logically true.

Decidability of the SQM

From the proof system, it is possible to decide the possibility of an agent being present at a location, at a certain time, if it is possible for that agent to be present at that location at the time t, given the antecedents using SQM. However, it is not possible to infer the fact that it is not possible for an agent to be present at a certain location and at a certain time. The reason is that, most of our axioms are implications and not equivalence. Thus, SQM is semi-decidable.

Conclusion

Sequel to the closure of sample proofs tableaux analysed in this work, the axioms that constitute the spatial qualification logical system are said to be the logical consequence of the set of formulae that make up the prior knowledge. Hence, the axioms that make up the SQM are proven to be logically true and sound for determining the possibility of an intelligent agent's spatial and temporal presence at location of incidence. Further study should focus on a wider context of quantified modal logics with constant domain of possible worlds to determine their decidability, soundness and completeness.

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