*Journal of Science Research (2013) Vol. 12: 351-360*

# On the Sensitivity of Probit and Complementary Log-Log Models to Violated Tolerance

**[1]Bolarinwa, I.A. and [2]Olubusoye, O.E.**

**Abstract**
In this article, the sensitivity of each of the fixed effects probit and complementary log-log models to gamma, Cauchy and lognormal tolerances was studied, through Monte Carlo experiments. Seven levels of sample size and four levels of number of time points were utilized for the simulation. Criteria used for comparison were the bias; variance and the root mean squared error. It was found that, typically for the probit and complementary log-log models, the lognormal tolerance produced the least absolute bias, while Cauchy produced the highest. Cauchy tolerance typically produced the least variance and lognormal tolerance, the highest for both the probit and the complementary log-log models. The lognormal tolerance produced the least variability around the true parameter value, and Cauchy, the highest for the probit model. For the complementary log-log model, gamma tolerance produced the least variability around the true parameter value and Cauchy, the largest. It was concluded that both the probit and the complementary log-log models were least sensitive to the lognormal tolerance, followed by gamma, and lastly, Cauchy tolerance.

**Key words:** Binary choice, Gamma, Cauchy, Lognormal, Link function.

## Introduction

Cases of modelling abound in various fields of human endeavour, ranging from socio-economic to medical sciences, where the dependent variable is a choice among a number of identified alternatives. That is, where the dependent variable assumes discrete values. Examples include occupational choice, choice of automobile, political party affiliation and HIV status of an individual. Unlike the case of continuous dependent variable that examines “how much?”, discrete choice models examine “which one?”. Of particular interest to this article is the case where choice is made between only two alternatives - the case of binary choice.

The use of discrete choice modelling is not limited to qualitative response variables alone. Sometimes, quantitative (continuous) response variables are classified into categories, which are assigned discrete values and then analyzed within the context of discrete choice modelling. Such remains valid much as the categorization has been done such that mutual exclusiveness among categories is maintained. For example, in an income study, income, though continuous, may be categorized into low, middle, and high. Such categorization of continuous variable into discrete form often results in ordered discrete choice models since, the categorization is often associated with some form of ordering (ranking). By assigning 1 to low income; 2 to middle income and 3 to high income, there is some form of ordering since those assigned 1 earn less than those assigned 2 and similarly those assigned 2 earn less than those assigned 3.

Various research works have been carried out in the area of choice modelling. [1] proposes a discrete-time dynamical system to model a class of binary choice games. [2] proposes a quadratic exponential type binary model and a $\sqrt{n}$ consistent conditional estimator. [3] suggests a generalized estimating equation approach to binary response panel data selection models. [4] presents for the index coefficients in a binary choice model, computationally simple root-*n*

**[1]Bolarinwa, I.A. and [2]Olubusoye, O.E.**
[1]*Department of Mathematics & Statistics, The Federal Polytechnic, Bida*
[2]*Department of Statistics, University of Ibadan, Ibadan*

ISSN 11179333

consistent and asymptotically normal estimators. [5] and [6] are among notable contributors to the literature on discrete choice modelling.

Parametric models whether cross-section or panel are known to be sensitive to their various parametric assumptions. The probit model for instance, requires that the latent model errors be standard normally distributed while, the complementary log-log requires standard type I extreme value (minimum) errors. The probability distribution that generates the latent model errors is known as *tolerance* in the literature. The probit hence, requires standard normal tolerance and the complementary log-log, standard type I value (minimum) tolerance. It is the focus of this article to assess the sensitivity of the fixed effects probit and complementary log-log models to (wrong) choices of the gamma, Cauchy and lognormal tolerances.

The plan of this article is as follows: Section 2 presents the modelling framework; Section 3, the methods; Section 4 presents the results and discussion while the last section concludes the article.

**Binary Choice Modelling Framework**

In the binary choice modelling framework, we shall view the discrete outcome ( $y$ ) as depending on whether a latent continuous variable (variable that is not directly measurable but inferred through another variable), $y^*$ crosses a threshold or not. We shall assume that $y^*$ is a linear function of a vector of explanatory variables, $x$.

Hence,

$$y^* = \beta^/ x + u \tag{1}$$

Instead of observing $y^*$, we are observing $y$ where,

$$y = \begin{cases} 1 & \text{if } y^* > 0 \\ 0 & \text{if } y^* \le 0 \end{cases} \tag{2}$$

$$P[y = 1/x] = E(y/x) \tag{3}$$

$$= F(\beta^/ x) \tag{4}$$

Eqn. (3) implies that the probability that the event occurs given $x$, usually denoted $p$, is the expected value of the response binary variable given $x$. The probability function that generates the residual error (tolerance), is the determinant of the link function and hence, the form of resulting binary choice model.

When the standard normal is selected (as tolerance), the resulting model is the probit model defined

$$p = \int_{-\infty}^{\beta^/ x} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du = \Phi(\beta^/ x) \tag{5}$$

Hence,

$$\Phi^{-}(p) = \beta^/ x. \tag{6}$$

The link is hence, the inverse of the standard cumulative normal distribution.

If the standard extreme value distribution is used as tolerance, we have the complementary log-log model,

$$p = \int_{-\infty}^{\beta^/ x} \exp(u)\exp(-\exp(u))du$$

$$= 1 - \exp(-\exp(\beta^/ x))$$

Hence,

$$\log(-\log(1-p)) = \beta^/ x \tag{7}$$

$\log(-\log(1-p))$ is therefore, the link. The model is linear in the log of the negative of the log of the compliment of $p$.

**Methods**

***The Model***

The model under consideration is a balanced one-way fixed effects error components model with a single covariate. This model is suitable for modelling binary panel data, when the omitted individual specific effect, is to be taken into cognizance and inference drawn applies only to the individuals involved in the study.

$$y_{it}^{*} = \beta x_{it} + \mu_i + v_{it} \quad i = 1,\dots, N;\ t = 1,\dots, T \qquad (8a)$$

where,

$$y_{it} = \begin{cases} 1 & \text{if } y_{it}^{*} > 0 \\ 0 & \text{if } y_{it}^{*} \le 0 \end{cases} \qquad (8b)$$

$y_{it}^{*}$ is a latent variable observed through $y_{it}$;

$\beta$ is a constant;

$x_{it}$ is the exogenous variable;

$\mu_i$ is the unobserved individual specific heterogeneity;

$v_{it}$ is the usual stochastic error term in regression.

***Data Generating Procedure***

The research adopted methods similar to those used in [7]. The latent and observed dependent variables were generated respectively, using eqn. (8a) and eqn. (8b). The exogenous variable $x_{it}$ utilised method similar to [8] method and it was generated as

$$x_{it} = 0.1t + 0.5x_{i,t-1} + \varepsilon_{it} \qquad (9)$$

where, $\varepsilon_{it}$ was uniformly distributed on the interval [-5, 5]. $x_{i0}$ was chosen as $5 + 10\varepsilon_{io}$. The [8] method was used in [9], [10], [11] and several others.

$v_{it}$ was generated as: Standard Cauchy; Gamma with parameters α = 1 and β = 1; and lognormal with parameters *m* = 1 and *s* = 0 .48. $\sigma_{\mu}^{2}$ and $\beta$ were each set at 1. N was set at 25, 50, 100, 150, 200, 250, and 300 while T was set at 5, 10, 15, and 20. The experiments were replicated 1000 times.

Each of the fixed effects probit and complementary log-log models was fitted to the simulated data. We hence had the following cases:

Case I: Probit modelling of standard Cauchy errors based data.

Case II: Probit modelling of gamma errors based data.

Case III: Probit modelling of lognormal errors based data.

Case IV: Complementary log-log modelling of standard Cauchy errors based data.

Case V: Complementary log-log modelling of gamma errors based data.

Case VI: Complementary log-log modelling of lognormal errors based data.

***Parameter Estimation***

Estimation of binary choice models is usually carried out via the maximum likelihood method, with the exception of the linear probability model.

To develop the framework, note that $y_{it}$ is Bernoulli with parameter $\beta^{/}x$ so that

$$f(y_{it}) = F(\beta^{/} x_{it})^{y_{it}} (1 - F(\beta^{/} x_{it}))^{1-y_{it}} \qquad (10a)$$

But, under the fixed effects framework, the likelihood function for NT observations is

$$L = \prod_{i=1}^{N}\prod_{t=1}^{T} F(\beta^{/} x_{it} + \mu_i)^{y_{it}} (1 - F(\beta^{/} x_{it} + \mu_i))^{1-y_{it}} \qquad (10b)$$

So that,

$$LogL = \sum_{i=1}^{N}\sum_{t=1}^{T}\left( y_{it} \log F(\beta^{/} x_{it} + \mu_i) + (1 - y_{it}) \log(1 - F(\beta^{/} x_{it} + \mu_i)\right) \qquad (10c)$$

and

$$\frac{\partial \log L}{\partial \beta} = \sum_{i=1}^{N}\sum_{t=1}^{T}\left[ \frac{(y_{it} - F(\beta^{/} x_{it} + \mu_i)}{F(\beta^{/} x_{it} + \mu_i)(1 - F(\beta^{/} x_{it} + \mu_i))} \right] F^{/}(\beta^{/} x_{it} + \mu_i) x_{it} = 0 \qquad (10d)$$

For the probit,

$$F^{/}(\beta^{/}x_{it}+\mu_i)=\frac{1}{\sqrt{2\pi}}e^{-\frac{(\beta^{/}x_{it}+\mu_i)^2}{2}} \quad (10e)$$

and for the complementary log-log,

$$F^{/}(\beta^{/}x_{it}+\mu_i)=\exp(\beta^{/}x_{it}+\mu_i)\exp(-\exp(\beta^{/}x_{it}+\mu_i)) \quad (10f)$$

No explicit (closed form) solution exists for the maximum likelihood estimator $\hat{\beta}_{MLE}$ as equations embedded in eqn. (10d) are non-linear in β except for the case of linear regression model. Iterative procedure is therefore required to obtain a solution. Since, the log likelihood for each of the probit and the complementary log-log is globally concave, the Hessian is automatically negative definite and the obtained optimum is therefore a maximum. The Newton-Raphson method was used in obtaining the estimates.

For the Newton's method,

$$\hat{\beta}^{(j)}=\hat{\beta}^{(j-1)}-\left(\frac{\partial^2\log L}{\partial\beta\partial\beta^{/}}\right)^{-1}_{\beta=\hat{\beta}^{(j-1)}}\left(\frac{\partial\log L}{\partial\beta}\right)_{\beta=\hat{\beta}^{(j-1)}} \quad (11)$$

where, $\hat{\beta}^{(j)}$ denotes the jth iterative solution. The inverse of the Hessian $\left(\frac{\partial^2\log L}{\partial\beta\partial\beta^{/}}\right)^{-1}$ is a direction matrix that reflects the curvature of the log likelihood, indicating how rapidly the gradient is changing while the gradient vector $\left(\frac{\partial\log L}{\partial\beta}\right)$ indicates the direction of the change in the log likelihood for a change in the parameters [11].

***Criteria for Performance Evaluation***

The performance criteria were the bias (BIAS), the variance (VAR) and the root mean square error (RMSE), defined below:

For the experiment that is replicated r times, let $\hat{\beta}_j$ represent the jth estimate of the true parameter value, $\beta$. Then,

$$BIAS(\hat{\beta})=\frac{1}{r}\sum_{j=1}^{r}(\hat{\beta}_j-\beta) \quad (12a)$$

$$VAR(\hat{\beta})=\frac{1}{r}\sum_{j=1}^{r}(\hat{\beta}_j-\bar{\hat{\beta}})^2 \quad (12b)$$

$$RMSE(\hat{\beta})=\left(\frac{1}{r}\sum_{j=1}^{r}(\hat{\beta}_j-\beta)^2\right)^{\frac{1}{2}} \quad (12c)$$

A two-way ANOVA in which tolerance (at 3 levels) and T (at 4 levels) were the factors, was performed to ascertain the significance or otherwise of the means of the biases across tolerance and across T.

**Results and Discussion**

The results are attached as Appendices I, II and III. Table 1 presents the biases for the three different tolerances. At N=25, gamma produced the maximum bias of 465.8% (at T=5) while the lognormal produced the least of 9.3% (at T=10) for the probit. For the complementary log-log, lognormal produced maximum bias of 89.2% (at T=5) and also the least of 8.5% (at T=20). At N=50, gamma produced the maximum bias of 72.4% (at T=5) while, lognormal produced the least of 20.6% (at T=10) for the probit; Cauchy produced the maximum of 68.9% (at T=20) and minimum bias of 5.2% (at T=20) for the complementary log-log. For values of N from 50 and above, Cauchy produced the largest bias and lognormal the least (at T=20) for the complementary log-log. For N=100 and above, for the probit, Cauchy produced the maximum (at T=20) and lognormal, the minimum (at T=5). At N=100, maximum bias of 67.7% and the minimum of 18.3% (at T=5) were obtained; for the complementary log-log, maximum bias was 69.2% and the minimum, 3.0%. For the probit, maximum bias of 67.6% and least of 22.2% were obtained for N=150 while maximum bias of 69.2% and minimum of 2.3% were obtained for the complementary log-log.

**Table 1: Biases**

| N | T | Probit | | | Complementary Log-Log | | |
|---|---|---|---|---|---|---|---|
| | | Tolerance | | | Tolerance | | |
| | | Gamma | Cauchy | LNormal | Gamma | Cauchy | LNormal |
| \25 | 5 | 4.65763 | -0.342297 | -.872134 | 0.498894 | -0.365708 | -.892164 |
| | 10 | -0.310444 | -0.638529 | -0.0926691 | -0.109233 | -0.587061 | 0.255171 |
| | 15 | -0.364068 | -0.65736 | -0.187657 | -0.20644 | -0.647128 | 0.122406 |
| | 20 | -0.367312 | -0.667107 | -0.195944 | -0.225597 | -0.67909 | 0.0848184 |
| 50 | 5 | -0.724429 | -0.613589 | 1.47568 | -0.614777 | -0.48279 | 0.481011 |
| | 10 | -0.369359 | -0.648793 | -0.206365 | -0.175952 | -0.602279 | 0.135949 |
| | 15 | -0.387173 | -0.66239 | -0.235987 | -0.226637 | -0.654557 | 0.0599323 |
| | 20 | -0.386852 | -0.673967 | -0.233097 | -0.244514 | -0.688863 | 0.0517009 |
| 100 | 5 | -0.32087 | -0.647831 | -0.183182 | -0.0408553 | -0.525067 | 0.208867 |
| | 10 | -0.388659 | -0.656887 | -0.245933 | -0.195317 | -0.611086 | 0.0907891 |
| | 15 | -0.397351 | -0.667267 | -0.261738 | -0.24106 | -0.659833 | 0.0349573 |
| | 20 | -0.391068 | -0.676238 | -0.255403 | -0.246854 | -0.692324 | 0.0295389 |
| 150 | 5 | -0.327817 | -0.657999 | -0.222292 | -0.0475293 | -0.534674 | 0.181796 |
| | 10 | -0.38977 | -0.661817 | -0.259716 | -0.195065 | -0.617461 | 0.084606 |
| | 15 | -0.404909 | -0.667447 | -0.26828 | -0.246872 | -0.661896 | 0.0330433 |
| | 20 | -0.399661 | -0.67624 | -0.261745 | -0.255071 | -0.691969 | 0.0229759 |
| 200 | 5 | -0.349692 | -0.667407 | -0.253814 | -0.0755957 | -0.546547 | 0.139673 |
| | 10 | -0.39784 | -0.661348 | -0.262949 | -0.203295 | -0.617423 | 0.077481 |
| | 15 | -0.403794 | -0.669991 | -0.272437 | -0.246344 | -0.664204 | 0.0293843 |
| | 20 | -0.402415 | -0.677038 | -0.262049 | -0.257346 | -0.692783 | 0.0244865 |
| 250 | 5 | -0.350405 | -0.666249 | -0.247803 | -0.0719312 | -0.545761 | 0.153716 |
| | 10 | -0.400056 | -0.664331 | -0.271917 | -0.207013 | -0.622409 | 0.0709003 |
| | 15 | -0.406208 | -0.669744 | -0.27824 | -0.247472 | -0.664164 | 0.024131 |
| | 20 | -0.401447 | -0.678284 | -0.269256 | -0.257519 | -0.694503 | 0.0192305 |
| 300 | 5 | -0.362494 | -0.671292 | -0.266031 | -0.086271 | -0.551988 | 0.136542 |
| | 10 | -0.39925 | -0.665498 | -0.272415 | -0.206177 | -0.622269 | 0.0758129 |
| | 15 | -0.405654 | -0.670039 | -0.276826 | -0.247554 | -0.664527 | 0.0304097 |
| | 20 | -0.402789 | -0.678717 | -0.267793 | -0.256857 | -0.694393 | 0.0181328 |

At N=200, the maximum bias of 67.7% and minimum of 25.4% were obtained for the probit while for the complementary log-log, maximum and minimum biases of 69.3% and 2.4% were respectively obtained. Maximum bias of 67.8% and minimum of 24.8% (at N=250) for the probit, while the maximum bias of 69.5% and the minimum of 1.9% were obtained for the complementary log-log. At N=300 for the probit, maximum bias of 67.9% and least of 26.6% were obtained. For the complementary log-log, maximum bias of

69.4% and the least of 1.8% were obtained. For values of N greater than 50, the maximum bias for the probit was quite stable (between 67.6, 72.4% for the probit and 69.2, and 69.4% for the complementary log-log). The bias was at most 465.8% for gamma; 67.9% for Cauchy and 87.2% for lognormal in the case of probit; 61.5% for gamma; 69.5% for Cauchy and 89.2% for lognormal in the case of complementary log-log.

Typically, bias decreased with increased T at any value of N. Only 2 (1 for each of gamma and lognormal) of the 84 biases were cases of overestimation while the rest were cases of underestimation for the probit. For the complementary log-log, gamma produced only a case of overestimation (at N=25, T=5); Cauchy, just as it did for the probit, underestimated all through; lognormal produced only a case of underestimation. In respect of sign of bias, the behaviour of gamma was complete opposite of that of lognormal. Table 2 presents the sum of ranks based on absolute bias. The lognormal tolerance typically produced the least absolute biases, followed by gamma, and lastly, Cauchy for both probit and complementary log-log.

**Table 2: Sum of Ranks based on Absolute Bias**

| | **Gamma** | **Cauchy** | **LogNormal** |
|---|---|---|---|
| PROBIT | 53 | 79 | 36 |
| CLOGLOG | 51 | 79 | 38 |

Tables 3 and 4 present the results of 2-way ANOVA in which tolerance (at 3 levels) and T (at 4 levels) were the factors. For the probit (see Table 3), no significant differences were observed for the biases across both the tolerance and T for N=25 and N=50. For N=100 and N=150, differences in mean biases were found to be significant across the tolerance and T. For values of N greater than 150, significant differences were observed for T, the contrary was observed for tolerance. At N=25 (see Table 4), for the complementary log-log, no significant differences were found for both tolerance and T. At N=50, no significant differences were found for tolerance, while significant differences were found for T. For N greater than 50, significant differences were found across both tolerance and T.

**Table 3: Summary of 2-Way ANOVA for Probit**

| **N** | **Factor** | **P-Value** | **Conclusion** |
|---|---|---|---|
| 25 | Tolerance | .556 | Do not reject $H_0$ |
| | T | .396 | Do not reject $H_0$ |
| 50 | Tolerance | .660 | Do not reject $H_0$ |
| | T | .144 | Do not reject $H_0$ |
| 100 | Tolerance | .012 | Reject $H_0$ |
| | T | .000 | Reject $H_0$ |
| 150 | Tolerance | .038 | Reject $H_0$ |
| | T | .000 | Reject $H_0$ |
| 200 | Tolerance | .202 | Do not reject $H_0$ |
| | T | .000 | Reject $H_0$ |
| 250 | Tolerance | .084 | Do not reject $H_0$ |
| | T | .000 | Reject $H_0$ |
| 300 | Tolerance | .317 | Do not reject $H_0$ |
| | T | .000 | Reject $H_0$ |

**Table 4: Summary of 2-Way ANOVA for Complementary log-log**

| N | Factor | P-Value | Conclusion |
|---|---|---|---|
| 25 | Tolerance | .985 | Do not reject $H_0$ |
| | T | .251 | Do not reject $H_0$ |
| 50 | Tolerance | .935 | Do not reject $H_0$ |
| | T | .005 | Reject $H_0$ |
| 100 | Tolerance | .000 | Reject $H_0$ |
| | T | .000 | Reject $H_0$ |
| 150 | Tolerance | .000 | Reject $H_0$ |
| | T | .000 | Reject $H_0$ |
| 200 | Tolerance | .000 | Reject $H_0$ |
| | T | .000 | Reject $H_0$ |
| 250 | Tolerance | .000 | Reject $H_0$ |
| | T | .000 | Reject $H_0$ |
| 300 | Tolerance | .000 | Reject $H_0$ |
| | T | .000 | Reject $H_0$ |

For the two models (see Table 5), typically, Cauchy produced the least variance, followed by gamma, and lastly, the lognormal, the only exception being the case N and T equal to 25 and 5 respectively. For all tolerances, diminished variance was associated with increased T. Unlike the variance, the root mean squared error did not exhibit a consistent behaviour in relation to either of the tolerance and T (see Table 6). However, Table 7 presents the sum of ranks due to each tolerance based on RMSE. Just as with the case of bias, the lognormal tolerance produced the least variability around the true parameter value, followed by gamma, and then, Cauchy for probit. For the complementary log-log, gamma tolerance produced the least, and Cauchy, the largest variability around the true parameter value.

On a general note, the lognormal tolerance can hence, be described as providing the least sensitivity and Cauchy, the highest sensitivity for both the probit and the complementary log-log models.

ISSN 11179333

**Table 5: Variances**

| N | T | Probit Tolerance Gamma | Probit Tolerance Cauchy | Probit Tolerance LNormal | Complementary Log-Log Tolerance Gamma | Complementary Log-Log Tolerance Cauchy | Complementary Log-Log Tolerance LNormal |
|---|---|---|---|---|---|---|---|
| 25 | 5 | 3992.235335 | 21.467634 | .29821 | 3.625756 | 0.543891 | .2913614 |
| | 10 | 0.235569 | 0.053098 | 0.574514 | 0.400969 | 0.073763 | 0.875347 |
| | 15 | 0.158843 | 0.043618 | 0.270344 | 0.251540 | 0.048831 | 0.517965 |
| | 20 | 0.149724 | 0.039870 | 0.251040 | 0.225161 | 0.038639 | 0.451622 |
| 50 | 5 | 0.198198 | 0.078427 | 561.732983 | 0.407944 | 0.145686 | 1.792800 |
| | 10 | 0.149302 | 0.045906 | 0.258173 | 0.255255 | 0.061857 | 0.518073 |
| | 15 | 0.135610 | 0.040106 | 0.219676 | 0.215756 | 0.043630 | 0.413498 |
| | 20 | 0.131917 | 0.036803 | 0.212757 | 0.199042 | 0.034219 | 0.395994 |
| 100 | 5 | 0.187528 | 0.052234 | 0.305836 | 0.371740 | 0.096562 | 0.631297 |
| | 10 | 0.133070 | 0.041673 | 0.209048 | 0.229193 | 0.055038 | 0.424925 |
| | 15 | 0.125390 | 0.038058 | 0.194661 | 0.198028 | 0.040652 | 0.376297 |
| | 20 | 0.127182 | 0.035683 | 0.192916 | 0.193858 | 0.032578 | 0.363869 |
| 150 | 5 | 0.173365 | 0.045637 | 0.249068 | 0.341729 | 0.085652 | 0.546853 |
| | 10 | 0.125722 | 0.040065 | 0.196124 | 0.217961 | 0.052465 | 0.412953 |
| | 15 | 0.121381 | 0.037613 | 0.187264 | 0.193199 | 0.039348 | 0.367050 |
| | 20 | 0.122748 | 0.035371 | 0.187915 | 0.188607 | 0.032291 | 0.355597 |
| 200 | 5 | 0.154161 | 0.041212 | 0.216460 | 0.309066 | 0.077421 | 0.479688 |
| | 10 | 0.124309 | 0.039615 | 0.191654 | 0.216773 | 0.051208 | 0.401478 |
| | 15 | 0.120229 | 0.036854 | 0.182705 | 0.191898 | 0.038484 | 0.361193 |
| | 20 | 0.120351 | 0.035000 | 0.187820 | 0.185548 | 0.031863 | 0.357960 |
| 250 | 5 | 0.152087 | 0.040233 | 0.217696 | 0.306747 | 0.075673 | 0.492168 |
| | 10 | 0.122708 | 0.038327 | 0.184196 | 0.213587 | 0.048930 | 0.391565 |
| | 15 | 0.119176 | 0.036861 | 0.179351 | 0.191014 | 0.038440 | 0.356835 |
| | 20 | 0.120810 | 0.034747 | 0.181922 | 0.185531 | 0.031506 | 0.351645 |
| 300 | 5 | 0.144691 | 0.038424 | 0.199614 | 0.295174 | 0.072217 | 0.461544 |
| | 10 | 0.123204 | 0.038120 | 0.184227 | 0.214215 | 0.048999 | 0.397018 |
| | 15 | 0.209414 | 0.036166 | 0.178569 | 0.190432 | 0.037902 | 0.358660 |
| | 20 | 0.119972 | 0.034642 | 0.181617 | 0.185457 | 0.031505 | 0.348423 |

**Table 6: Root Mean Squared Errors**

| N | T | Probit Tolerance Gamma | Probit Tolerance Cauchy | Probit Tolerance LNormal | Complementary Log-Log Tolerance Gamma | Complementary Log-Log Tolerance Cauchy | Complementary Log-Log Tolerance LNormal |
|---|---|---|---|---|---|---|---|
| 25 | 5 | 63.355574 | 4.645945 | 1.162138 | 1.968413 | 0.823185 | 1.2183161 |
| | 10 | 0.576146 | 0.678835 | 0.763611 | 0.642574 | 0.646841 | 0.969773 |
| | 15 | 0.539805 | 0.689739 | 0.552774 | 0.542363 | 0.683816 | 0.730033 |
| | 20 | 0.533519 | 0.696349 | 0.537991 | 0.525410 | 0.706968 | 0.677360 |
| 50 | 5 | 0.850292 | 0.674477 | 23.746802 | 0.886507 | 0.615445 | 1.422734 |
| | 10 | 0.534535 | 0.683256 | 0.548416 | 0.534989 | 0.651611 | 0.732499 |
| | 15 | 0.534334 | 0.692001 | 0.524753 | 0.516837 | 0.687078 | 0.645825 |
| | 20 | 0.530633 | 0.700738 | 0.516809 | 0.508752 | 0.713268 | 0.631400 |
| 100 | 5 | 0.538967 | 0.686963 | 0.582573 | 0.611072 | 0.610129 | 0.821537 |
| | 10 | 0.533034 | 0.687876 | 0.519164 | 0.517051 | 0.654572 | 0.658155 |
| | 15 | 0.532238 | 0.695200 | 0.512999 | 0.506101 | 0.689951 | 0.614425 |
| | 20 | 0.533063 | 0.702126 | 0.508081 | 0.501420 | 0.715465 | 0.603938 |
| 150 | 5 | 0.529933 | 0.691809 | 0.546334 | 0.586505 | 0.609531 | 0.761514 |
| | 10 | 0.532051 | 0.683256 | 0.513397 | 0.509812 | 0.651611 | 0.648160 |
| | 15 | 0.534165 | 0.692001 | 0.509154 | 0.504127 | 0.687078 | 0.606747 |
| | 20 | 0.531486 | 0.700738 | 0.506385 | 0.503655 | 0.713268 | 0.596762 |
| 200 | 5 | 0.525781 | 0.697599 | 0.529982 | 0.561053 | 0.613299 | 0.706538 |
| | 10 | 0.531588 | 0.690649 | 0.510682 | 0.508037 | 0.657586 | 0.638343 |
| | 15 | 0.532239 | 0.696952 | 0.506880 | 0.502576 | 0.692568 | 0.601712 |
| | 20 | 0.531309 | 0.702410 | 0.506448 | 0.501772 | 0.715410 | 0.598798 |
| 250 | 5 | 0.524281 | 0.695788 | 0.528302 | 0.558499 | 0.611169 | 0.718190 |
| | 10 | 0.531745 | 0.692576 | 0.508070 | 0.506400 | 0.660547 | 0.629755 |
| | 15 | 0.533086 | 0.696719 | 0.506723 | 0.502251 | 0.703432 | 0.597844 |
| | 20 | 0.531009 | 0.703432 | 0.504401 | 0.501843 | 0.716827 | 0.593308 |
| 300 | 5 | 0.525446 | 0.699326 | 0.519987 | 0.550106 | 0.613928 | 0.692955 |
| | 10 | 0.531606 | 0.693547 | 0.508367 | 0.506680 | 0.660467 | 0.634638 |
| | 15 | 0.532699 | 0.696820 | 0.505175 | 0.501713 | 0.692615 | 0.599654 |
| | 20 | 0.531236 | 0.703774 | 0.503319 | 0.501430 | 0.716719 | 0.590552 |

**Table 7: Sum of Ranks Based on RMSE**

| | Gamma | Cauchy | LogNormal |
|---|---|---|---|
| PROBIT | 48 | 80 | 40 |
| CLOGLOG | 32 | 71 | 65 |

## Conclusions

The sensitivity of each of the fixed effects probit and complementary log-log models to three forms of tolerance has been investigated. We found that the two models were least sensitive to the lognormal tolerance, followed by gamma, and lastly, Cauchy; the lognormal exhibited the least variability around the true parameter value, followed by gamma, and then, Cauchy for probit while for the complementary log-log, gamma exhibited the least and Cauchy, the largest.

## References


(1) Bischi, G. and Merlone, U. 2009. Global dynamics in binary choice models with social influence. *Journal of Mathematical Sociology*, 33: 277-302.

(2) Bartolucci, F. and Nigro, V. 2010. *A dynamic model for binary panel data with unobserved heterogeneity admitting a* $\sqrt{n}$*-consistent conditional estimator* (CEIS Research Paper No. 97).

(3) Spiess, M. and Kroh, M. 2010. A selection problem for panel data: The prospects of green party support. *Political Analysis*, 18(*2*): 172-188.

(4) Yildiz, N. 2013. Estimation of binary choice models with linear index and dummy endogenous variables. *Econometric Theory*, 29(*2*): 354-392.

(5) Hsiao, C. 2003. Analysis of panel data (2nd ed.). Cambridge: Cambridge University Press.

(6) Baltagi, B.H. 2008. Econometric analysis of panel data (4th ed.). Chichester: John Wiley.

(7) Heckman, J.J. 1981. Statistical models for discrete data, in Hsiao, C. (2003). Analysis of panel data (2nd ed.). Cambridge: Cambridge University Press.

(8) Nerlove, M. 1971. Further evidence on the estimation of dynamic economic relations from a time series of cross sections. *Econometrica*, 39, 383-396.

(9) Olubusoye, O.E. and Bolarinwa, I.A. 2006a. A Comparative study of alternative estimators for variance components models with serially correlated errors. *Science Focus - An International Journal of Biological and Physical Sciences*, 11(1): 10-14.

(10) Olubusoye, O.E. and Bolarinwa, I.A. 2006b. An investigation into the performance of estimators of error Components models in the presence of stationary auto-regressive error components. *Journal of Scientific and Industrial Studies*, 4(*2*): 77-82.

(11) Bolarinwa, I.A. and Olubusoye, O.E. 2008. On sensitivity of some estimators of error component models to nonstationarity of autoregression in error components. *International Journal of Physical Science*, 3(*2*): 93-98.

(12) Long, J.S. 1997. Regression models for categorical and limited dependent variables. California: SAGE.


ISSN 11179333