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Cholesky Decomposition of Variance-Covariance Matrix Effect on the Estimators of Seemingly Unrelated Regression Model

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Abstract

Seemingly Unrelated Regression (SUR) model which takes cognizance of correlations and strength of association between the error variables to yield more efficient estimates is a common tool in multi-equation regression analysis. The use of Cholesky method to partition the variance-covariance matrix $\sum$ into lower and upper triangular matrices to establish contemporaneous relationship among equations through their errors was investigated in this paper. Literature on the efficiency of SUR and Ordinary Least Squares (OLS) estimators assumed inconsequential the use of either upper or lower triangular matrices from a decomposed variance-covariance matrix. This study investigated the sensitivity of the two triangular matrices on SUR and OLS estimators. A Monte Carlo experiment was performed on a four-equation model with sample sizes n = 10, 30, 50, 100, 500 and 1000 and each replicated 10000 times. The Average Mean Square Error (AMSE) was used to assess the performance of the estimators. It was observed that the upper triangular matrix had higher AMSE values than the lower triangular matrix for SUR and OLS estimators. Also, the AMSE of SUR estimator was lower than that of OLS estimator, irrespective of the triangular matrix used.

Key words: Triangular matrices, Average Mean Square Error, Ordinary Least Squares, Seemingly Unrelated Regression.

Introduction

The Seemingly Unrelated Regression (SUR), which considers a joint modelling, is a special case of the multivariate regression model [21, 22]. It is used to capture the effect of different covariates allowed in the regression equations. Seemingly unrelated regressions allow the estimation of multiple models simultaneously while accounting for the correlated errors. The contemporaneous correlation, which could account for some common unnoticeable or unquantifiable effect, which the disturbances of several separate regression equations are expected to reflect is the correlation between disturbances in different equations. Taking cognizance of such correlation leads to efficient estimates of the coefficients and standard errors. The variance-covariance matrix is a Hermittan positive-definite non-singular symmetric matrix usually decomposed by the Cholesky decomposition. The Cholesky decomposition partitions the variance-covariance matrix into an upper and lower triangular matrix [7, 21, 22].

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The main motivation for use of SUR is to gain efficiency in estimation by combining information on different equations. Efficiency would be attained when contemporaneous correlation between the disturbances is high and explanatory variables in different equations are uncorrelated. Definite gains are obtained for all sample sizes when $|\rho| > 0.3$ where, ρ is the contemporaneous correlation for the disturbances in the equations [1, 5, 6, 7, 13, 14, 19, 21]. There is no clear-cut distinction between SUR and univariate models (GLS and OLS are identical) when the same set or subset (and values) of covariates, which may not likely lead to more efficient estimates in SUR are used rather than running the model separately, which supports the findings of [3, 8]. Within the Bayesian context, [1] investigated how large the contemporaneous correlations among disturbances should be in order for SUR to be more efficient than OLS. The study asserts that definite gains are obtained when $|\rho| > 0.333$ which compares well with [4] who used a frequentist approach. [20] examined the relative gain/loss in efficiency of SUR estimators when one or more pair

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of the predictors in the system of equations is correlated (non-orthogonal).

The parametric SUR was extended to nonparametric, semiparametric and geo-additive SUR model [11, 12, 16]. Recent studies on SUR [2, 9, 18]. The major gap observed in all these scholarly works is the assumed inconsequential application of decomposed variance-covariance matrix on the SUR estimates. To address this, an arbitrarily chosen four-equation model with the true covariance given by [1] on small and large sample sizes (n= 10, 30, 50, 100, 500 & 1000) replicated 10000 times in turn was investigated. The rest of this paper discusses the model and the simulation scenarios in section 2. Section 3 gives the presentation and discussion of the results. Section 4 concludes the paper.

The Model

Consider the SURE model consisting of stacked system of M equations:

$$Y = X\beta + \varepsilon \tag{1}$$

where

$$\underset{mn\times 1}{\begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix}} = \underset{mn\times \sum k_i}{\begin{bmatrix} X_1 & \cdots & 0 \\ \vdots & & \\ 0 & \cdots & X_m \end{bmatrix}} \underset{\sum k_i \times 1}{\begin{bmatrix} \beta_1 \\ \vdots \\ \beta_m \end{bmatrix}} + \underset{mn\times 1}{\begin{bmatrix} \varepsilon_1 \\ \vdots \\ \varepsilon_m \end{bmatrix}} \tag{2}$$

y_i is an $mn\times 1$ vector of observations on the i^{th} response variable

X_i is an $mn \times \sum k_i$ matrix of explanatory variables

β_i is a $\sum k_i \times 1$ vector of regression parameters

ε_i is $mn\times 1$ vector of disturbances

The variance-covariance matrix of the disturbance in (2) is

$$E(\varepsilon\varepsilon') = \sum \otimes I_n, \quad \text{where,} \quad E(\varepsilon) = 0 \tag{3}$$

That is, the expectation is assumed to be zero at different equation and different sample point. The positive definite $m\times m$ matrix $\sum$ is given as

$$\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots\cdots & \sigma_{1M} \\ \sigma_{21} & \sigma_{22} & \cdots\cdots & \sigma_{2M} \\ . & & & \\ . & & & \\ \sigma_{M1} & \sigma_{M2} & \cdots\cdots & \sigma_{MM} \end{bmatrix} \otimes I_n \tag{4}$$

So that,

$$\Omega = \Sigma \otimes I_n \tag{5}$$

Its inverse is given as

$$\Omega^{-1} = \Sigma^{-1} \otimes I_n \tag{6}$$

The OLS is used to estimate one equation at a time since each equation is a classical regression. The Generalized Least Squares (GLS) estimator is used to estimate an $m\times m$ covariance matrix of the disturbances Σ. The common estimators of β for classical linear SUR model [17] usually are:

(i) OLS: This is the equation-by-equation OLS estimator.

$$\hat{\beta}_{ols} = (\sum_{i=1}^{n} \tilde{X}_i \tilde{X}_i')^{-1} \sum_{i=1}^{n} \tilde{X}_i Y_i \tag{7}$$

(ii) GLS: If $\sum$ is known $, |\rho| > 0.3$, GLS estimator is

$$\hat{\beta}_{GLS} = (X'\hat{\Omega}^{-1}X)^{-1}X'\Omega^{-1}y = (X'(\hat{\Sigma}^{-1}\otimes I)X)^{-1}X'(\hat{\Sigma}^{-1}\otimes I)y \tag{8}$$

When $|\rho|;\ 0$, OLS is preferred while GLS also known as Zellner's Least Squares Estimator (ZLSE) is more efficient than OLSE when $|\rho| \geq 0.3$ [10, 15]

where

$$\rho = \frac{\sigma_{ij}}{\sqrt{\sigma_{ii}\sigma_{jj}}}, \quad \sigma_{ij} \neq 0 \tag{9}$$

The Design of the Simulation Experiment

An arbitrary four-equation SUR model with correlated errors was specified as follows:

$$\begin{aligned} y_1 &= \beta_{10} + \beta_{11}X_{11} + \beta_{12}X_{12} + \beta_{13}X_{13} + \varepsilon_1 \\ y_2 &= \beta_{20} + \beta_{21}X_{21} + \beta_{22}X_{22} + \varepsilon_2 \\ y_3 &= \beta_{30} + \beta_{31}X_{31} + \varepsilon_3 \\ y_4 &= \beta_{40} + \beta_{41}X_{41} + \varepsilon_4 \end{aligned} \qquad (10)$$

The positive definite 4×4 variance-covariance matrix is defined by

$$\Sigma = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} \end{pmatrix} \qquad (11)$$

The data series were generated using the following steps.

Step 1. The vectors of $X's$ of independent parametric regressors were generated. ***x11** and **x12*** are drawn from normal distribution and ***x13**, **x21**, **x22**, **x31*** and ***x41*** are drawn from uniform distribution U(-3, 3)

Step 2. Four mutually independent $N(0,1)$ sequences $(\varepsilon_{i1}, \varepsilon_{i2}, \varepsilon_{i3}, \varepsilon_{i4})$ were generated and transformed to ensure that the disturbance terms are contemporaneously correlated and distributed as $N(0, \Sigma)$. Le t $\varepsilon = (\varepsilon_{i1}, \varepsilon_{i2}, \varepsilon_{i3}, \varepsilon_{i4})'$, where, $E(\varepsilon) = 0$.

Since, Σ by definition is a positive definite matrix, there exist a non-singular triangular matrix P such that

$$\Sigma = PP' \qquad (12)$$

Therefore, the random vectors are

$$U = P'\varepsilon \qquad (13)$$

By construction, the vectors

$E(\varepsilon_i) = 0$ where,

$\Sigma = C\,\text{ov}(\varepsilon\varepsilon') = \Omega \otimes I_n$

$\otimes$ is the Kronecker product. (14)

Since Σ is a positive- definite matrix, we decomposed the matrix such that $\Sigma = PP'$

The upper triangular matrix is

$$P = \begin{pmatrix} p^*_{11} & p^*_{12} & p^*_{13} & p^*_{14} \\ 0 & p^*_{22} & p^*_{23} & p^*_{24} \\ 0 & 0 & p^*_{33} & p^*_{34} \\ 0 & 0 & 0 & p^*_{44} \end{pmatrix} \qquad (15)$$

While the lower triangular matrix is

$$P' = \begin{pmatrix} p^*_{11} & 0 & 0 & 0 \\ p^*_{21} & p^*_{22} & 0 & 0 \\ p^*_{31} & p^*_{32} & p^*_{33} & 0 \\ p^*_{33} & p^*_{33} & p^*_{33} & p^*_{44} \end{pmatrix} \qquad (16)$$

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Therefore,

$$PP^{'} = \Sigma = \begin{pmatrix} p_{11}^{*} & p_{12}^{*} & p_{13}^{*} & p_{14}^{*} \\ 0 & p_{22}^{*} & p_{23}^{*} & p_{24}^{*} \\ 0 & 0 & p_{33}^{*} & p_{34}^{*} \\ 0 & 0 & 0 & p_{44}^{*} \end{pmatrix} \begin{pmatrix} p_{11}^{*} & 0 & 0 & 0 \\ p_{21}^{*} & p_{22}^{*} & 0 & 0 \\ p_{31}^{*} & p_{32}^{*} & p_{33}^{*} & 0 \\ p_{33}^{*} & p_{33}^{*} & p_{33}^{*} & p_{44}^{*} \end{pmatrix} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \sigma_{24} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \sigma_{34} \\ \sigma_{41} & \sigma_{42} & \sigma_{43} & \sigma_{44} \end{pmatrix} = \begin{pmatrix} 1 & 0.7 & 0.6 & 0.9 \\ & 1 & 0.7 & 0.9 \\ & & 1 & 0.7 \\ & & & 1 \end{pmatrix} \quad (17)$$

From (17), we obtained

$$\begin{aligned} \rho_{11}^{*2} + \rho_{12}^{*2} + \rho_{13}^{*2} + \rho_{14}^{*2} &= 1 \\ \rho_{12}^{*}\rho_{22}^{*} + \rho_{13}^{*}\rho_{23}^{*} + \rho_{14}^{*}\rho_{24}^{*} &= 0.7 \\ \rho_{13}^{*}\rho_{33}^{*} + \rho_{14}^{*}\rho_{34}^{*} &= 0.6 \\ \rho_{14}^{*}\rho_{44}^{*} &= 0.9 \\ &. \\ &. \\ &. \\ \rho_{44}^{*2} &= 1 \end{aligned} \quad (18)$$

We obtained the other values by using (19) for (18)

$$\rho_{44}^{*2} = 1$$

$$\rho_{44}^{*} = \sqrt{1} = 1 \quad (19)$$

Decomposing the variance-covariance matrix, gave

$$\mathrm{P} = \begin{pmatrix} 0.35509 & -0.249292 & -0.04201 & 0.9 \\ 0 & 0.42473 & 0.09802 & 0.9 \\ 0 & 0 & 0.71414 & 0.7 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \mathrm{P}' = \begin{pmatrix} 0.35509 & 0 & 0 & 0 \\ -0.249292 & 0.042473 & 0 & 0 \\ -0.04201 & 0.09802 & 0.71414 & 0 \\ 0.9 & 0.9 & 0.7 & 1 \end{pmatrix} \quad (20)$$

The four random disturbance series are formed using

$$U = P^{'}\varepsilon \quad (21)$$

The random disturbance series for the upper triangular matrix is

$$\begin{pmatrix} u_{1i} \\ u_{2i} \\ u_{3i} \\ u_{4i} \end{pmatrix} = \begin{pmatrix} 0.35509 & -0.249292 & -0.04201 & 0.9 \\ & 0.04247 & 0.09802 & 0.9 \\ & & 0.71414 & 0.7 \\ & & & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \varepsilon_{3i} \\ \varepsilon_{4i} \end{pmatrix} \quad (22)$$

$$\begin{aligned} u_{1i} &= 0.35509\varepsilon_{1i} - 0.249292\varepsilon_{2i} - 0.04201\varepsilon_{3i} + 0.9\varepsilon_{4i} \\ u_{2i} &= 0.42473\varepsilon_{2i} + 0.09802\varepsilon_{3i} + 0.9\varepsilon_{4i} \\ u_{3i} &= 0.71414\varepsilon_{3i} + 0.7\varepsilon_{4i} \\ u_{4i} &= \varepsilon_{4i} \end{aligned} \quad (23)$$

While the random disturbance series for the lower triangular matrix is

$$\begin{pmatrix} u_{1i} \\ u_{2i} \\ u_{3i} \\ u_{4i} \end{pmatrix} = \begin{pmatrix} 0.35509 & 0 & 0 & 0 \\ -0.249292 & 0.042473 & 0 & 0 \\ -0.04201 & 0.09802 & 0.71414 & 0 \\ 0.9 & 0.9 & 0.7 & 1 \end{pmatrix} \begin{pmatrix} \varepsilon_{1i} \\ \varepsilon_{2i} \\ \varepsilon_{3i} \\ \varepsilon_{4i} \end{pmatrix} \tag{24}$$

$$\begin{aligned} u_{1i} &= 0.35509\varepsilon_{1i} \\ u_{2i} &= -0.249292\varepsilon_{1i} + 0.42473\varepsilon_{2i} \\ u_{3i} &= -0.04201\varepsilon_{1i} + 0.09802\varepsilon_{2i} + 0.71414\varepsilon_{3i} \\ u_{4i} &= 0.9\varepsilon_{1i} + 0.9\varepsilon_{2i} + 0.7\varepsilon_{3i} + \varepsilon_{4i} \end{aligned} \tag{25}$$

In this manner, the desired error terms were obtained.

Step 3. Specific values were assigned to the structural parameters. The coefficients are pre-determined as the convention is in Monte-Carlo experiment

$$y_1 = 0.4 + 0.5x_{11} + 0.7x_{12} + 0.1x_{13} + \varepsilon_1$$
$$y_2 = 0.5 + 0.7x_{21} - 0.2x_{22} + \varepsilon_2$$
$$y_3 = -1.1 + 0.9x_{31} + \varepsilon_3$$
$$y_4 = 0.2 - 1.3x_{41} + \varepsilon_4$$

Step 4. For sample size n= 10, 30, 50 100, 500, 1000, the experiment was replicated 10000 times.

Step 5. We estimated the known parameters as if they were not known using SUR and OLS estimation methods, we then compared the performance of the estimators.

Codes were written using the STATA Do-file editor and the steps 1- 5 specified above were all executed using STATA software package, 8.0 version.

Analysis and Discussion of Simulation Results

Presentation of Results

The results of the AMSE and Mean of OLS and SUR estimators were presented using the two triangular matrices. The AMSE values of upper and lower triangular matrices for SUR when n = 10, are: 0.0658, and 0.0950, 0.0073 and 0.0217, 0.0051 and 0.0034 while OLS estimators are given as 0.3124 and 0.4649, 0.0333 and 0.0351, 0.0135 and 0.0093 respectively. When n = 30, the SUR AMSE values of upper and lower triangular matrices are: 0.0589 and 0.0559, 0.0024 and 0.0117, 0.0111 and 0.0005 while OLS estimators are given as 0.2725 and 0.2589, 0.0111 and 0.0360, 0.0342 and 0.0015. Similar results were obtained when n= 50 for the SUR AMSE values of the upper and lower triangular matrices which are given as: 0.0234 and 0.0261, 0.0013 and 0.0117, 0.0111 and 0.0005 while OLS estimators are 0.1192 and 0.1336, 0.0065 and 0.0360, 0.0342 and 0.0015. In the same vein, we had 0.0087 and 0.0092, 0.0006 and 0.0020, 0.0021 and 0.0001 for the upper and lower triangular matrices SUR AMSE values while for OLS 0.0540 and 0.0528, 0.0022 and 0.0066, 0.0070 and 0.0004 when n= 100.

Also, the SUR AMSE for the upper triangular matrix and lower triangular matrix when n = 1000 is given as 0.0006 and 0.0007, 0.0003 and 0.0002, 0.0002 and 0.0000 while we had 0.0049 and 0.0052, 0.0003 and 0.0006, 0.0007 and 0.0000 for the OLS estimators (see Tables 1 and 2). The mean of the estimators replicated 10000 in turn for the two triangular matrices for both OLS and SUR are presented in Tables 3 and 4.

Table 1: AMSE from Upper Triangular Matrix

n	Estimators	β_{10}	β_{11}	β_{12}	β_{13}	β_{20}	β_{21}	β_{22}	β_{30}	β_{31}	β_{40}	β_{41}
10	**SUR**	0.0556	0.0658	0.0950	0.0073	0.0578	0.0102	0.0069	0.0766	0.01368	0.0570	0.0112
	OLS	0.0969	0.3124	0.4649	0.0333	0.0977	0.0375	0.0258	0.0994	0.0352	0.08101	0.0430
30	**SUR**	0.0284	0.0589	0.0559	0.0024	0.0299	0.0019	0.0024	0.0282	0.0044	0.0308	0.0010
	OLS	0.0334	0.2725	0.2589	0.0111	0.0364	0.0122	0.0097	0.0305	0.0105	0.0331	0.0096
50	**SUR**	0.0183	0.0234	0.0261	0.0013	0.0194	0.0012	0.0015	0.0187	0.0020	0.0196	0.0007
	OLS	0.0202	0.1192	0.1336	0.0065	0.0212	0.0087	0.0069	0.0195	0.0073	0.0229	0.0073
100	**SUR**	0.0096	0.0087	0.0092	0.0006	0.0097	0.0005	0.0006	0.0097	0.0015	0.0097	0.0002
	OLS	0.0102	0.0520	0.0548	0.0032	0.0100	0.0029	0.0036	0.0099	0.0032	0.0100	0.0032
500	**SUR**	0.0020	0.0013	0.0013	0.0001	0.0020	0.0001	0.0001	0.0020	0.0003	0.0020	0.0000
	OLS	0.0020	0.0096	0.0094	0.0007	0.0020	0.0007	0.0007	0.0020	0.0006	0.0020	0.0006
1000	**SUR**	0.0010	0.0006	0.0007	0.0000	0.0010	0.0000	0.0000	0.0010	0.0002	0.0010	0.0000
	OLS	0.0010	0.0049	0.0052	0.0003	0.0010	0.0003	0.0003	0.0010	0.0003	0.0010	0.0003

Table 2: AMSE from Lower Triangular Matrix

n	Estimators	β_{10}	β_{11}	β_{12}	β_{13}	β_{20}	β_{21}	β_{22}	β_{30}	β_{31}	β_{40}	β_{41}
10	**SUR**	0.0062	0.0217	0.0051	0.0034	0.0110	0.0447	0.0093	0.0159	0.2633	0.0012	0.0730
	OLS	0.0107	0.0351	0.0135	0.0093	0.0345	0.0578	0.0204	0.0513	0. 3631	0.0037	0.1927
30	**SUR**	0.0038	0.0117	0.0111	0.0005	0.0079	0.0012	0.0015	0.0142	0.0034	0.0909	0.0113
	OLS	0.0044	0.0360	0.0342	0.0015	0.0092	0.0025	0.0031	0.1527	0.0053	0.0976	0.0283
50	**SUR**	0.0038	0.0117	0.0111	0.0005	0.0079	0.0012	0.0015	0.0142	0.0034	0.0909	0.0113
	OLS	0.0044	0.0360	0.0342	0.0015	0.0092	0.0025	0.0031	0.1527	0.0053	0.0976	0.0283
100	**SUR**	0.0012	0.0020	0.0021	0.0001	0.0024	0.0003	0.0004	0.0051	0.0011	0.0305	0.0034
	OLS	0.0013	0.0066	0.0070	0.0004	0.0025	0.0007	0.0008	0.0052	0.0017	0.0312	0.0098
500	**SUR**	0.0003	0.0004	0.0004	0.0000	0.0005	0.0001	0.0001	0.0010	0.0002	0.0062	0.0006
	OLS	0.0003	0.0012	0.0012	0.0001	0.0005	0.0002	0.0002	0.0011	0.0003	0.0063	0.0020
1000	SUR	0.0001	0.0002	0.0002	0.0000	0.0002	0.0000	0.0000	0.0005	0.0001	0.0031	0.0003
	OLS	0.0001	0.0006	0.0007	0.0000	0.0002	0.0001	0.0001	0.0005	0.0002	0.0031	0.0010

Table 3: Upper Triangular Matrix

	SUR MEAN						OLS MEAN					
	10	**30**	**50**	**100**	**500**	**1000**	**10**	**30**	**50**	**100**	**500**	**1000**
y_1												
β_{10}	0.3936	0.3719	0.3668	0.4043	0.4025	0.3999	0.4049	0.3786	0.3661	0.4037	0.4026	0.3999
β_{11}	0.4463	0.4835	0.5088	0.4952	0.5007	0.5011	0.4515	0.3469	0.5374	0.4723	0.4976	0.4995
β_{12}	0.7094	0.6891	0.7238	0.6878	0.7010	0.7011	0.7568	0.5794	0.7591	0.6560	0.7012	0.6980
β_{13}	0.0798	0.1053	0.0950	0.1068	0.1003	0.0999	0.0707	0.1304	0.0973	0.1099	0.0998	0.1000
y_2												
β_{20}	0.6454	0.4769	0.4679	0.4942	0.5018	0.5007	0.6449	0.4843	0.4678	0.4946	0.5019	0.5007
β_{21}	0.7864	0.6811	0.7020	0.6989	0.6998	0.6999	0.7890	0.6962	0.7013	0.6934	0.6992	0.6995
β_{22}	-0.1903	-0.1884	-0.1988	-0.1993	-0.2005	-0.1996	-0.1812	-1.1930	-0.1989	-0.2040	-0.1998	-0.1994
y_3												
β_{30}	-1.1167	-1.1144	-1.1117	-1.0939	-1.1013	-1.0989	-1.0939	-1.1150	-1.1121	-1.0939	-1.1012	-1.0989
β_{31}	1.2306	1.2903	1.2806	1.2995	1.2980	1.2997	1.2794	1.2932	1.2734	1.3012	1.2985	1.2997
y_4												
β_{40}	0.1962	0.1715	0.1597	0.2005	0.2023	0.2002	0.1447	0.1743	0.1414	0.1998	0.2022	0.2002
β_{41}	-1.3479	- 1.2975	-1.2806	-1.2999	-1.3000	-1.3001	-1.4370	-1.2703	-1.2684	-1.2946	-1.3013	-1.3001

Table 4: Lower Triangular Matrix

	SUR MEAN						OLS MEAN					
	10	**30**	**50**	**100**	**500**	**1000**	**10**	**30**	**50**	**100**	**500**	**1000**
y_1												
β_{10}	0.4074	0.4032	0.4017	0.4009	0.4003	0.4000	0.4098	0.4036	0.4031	0.3999	0.4003	0.4000
β_{11}	0.5330	0.5515	0.4988	0.5033	0.5006	0.5008	0.5507	0.5489	0.4914	0.4993	0.5007	0.5011
β_{12}	0.8005	0.7398	0.6981	0.7028	0.7001	0.7010	0.8278	0.7359	0.7009	0.6931	0.7010	0.7011
β_{13}	0.1098	0.0918	0.0994	0.1020	0.1002	0.0993	0.1199	0.0936	0.0971	0.1062	0.1004	0.0999
y_2												
β_{20}	0.5367	0.5067	0.4994	0.4922	0.4999	0.5004	0.5361	0.5082	0.5002	0.4927	0.4999	0.5004
β_{21}	0.6603	0.6889	0.6982	0.6975	0.7002	0.7001	0.6530	0.6883	0.7003	0.6931	0.7003	0.7000
β_{22}	-0.2350	-0.2009	-0.1985	-0.1986	-0.2006	-0.1968	-0.2535	-0.2046	-0.1964	-0.1995	-0.2006	-0.1995
y_3												
β_{30}	-1.1243	-1.0918	-1.0840	-1.0961	-1.1029	-1.0989	-1.1128	-1.0921	-1.0841	-1.0961	-1.1029	-1.0989
β_{31}	1.2216	1.2915	1.2849	1.2984	1.2984	1.2999	1.2460	1.2928	1.2844	1.2978	1.2982	1.2997
y_4												
β_{40}	0.1815	0.2219	0.1572	0.1936	0.2009	0.2021	0.1842	0.2229	0.1510	0.1940	0.2009	0.2022
β_{41}	-1.2216	1.2721	-1.2589	-1.2946	-1.3014	-1.3000	-1.5195	-1.2629	-1.2488	-1.2979	-1.3020	-1.2985

Discussion of Results

It was observed that the AMSE of the lower triangular matrix are lower than that of the upper triangular matrix for both SUR and OLS estimators except for β_{40} and β_{41} in equation y_4 where we had a reverse order. The AMSE of SUR estimators were lower than the OLS estimators which depict that SUR estimator performed better than OLS estimator. Another salient result is that, as the sample size increases, the AMSE values reduced considerably. Comparing the means of the upper triangular matrix and lower triangular matrix of SUR and OLS estimators, it can be observed that the larger the sample size, the closer the mean of the parameters to its true population parameters.

Conclusion

This study found that the lower triangular matrix gave smaller AMSE when compared with the AMSE of the upper triangular matrix of the decomposed variance-covariance matrix. The lower triangular matrix of the decomposed variance-covariance matrix is adjudged the best for further analysis.

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