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Forecasting in One-Dimensional and Generalized Integrated Autoregressive Moving Average Bilinear Time Series Models

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Abstract
In this paper, forecast of one-dimensional integrated autoregressive moving average bilinear time series model is compared with forecast of generalized integrated autoregressive moving average bilinear time series model. We described methods for estimation of these models and the forecast. It is also pointed out that for this class of non-linear time series models; it is possible to obtain optimal forecast. The estimation technique is illustrated with respect to a time series, and the optimal forecast of these time series are calculated. A comparison of these forecasts is made using the two models under study. The mean square error for forecast in one-dimensional integrated autoregressive moving average bilinear model is smaller than the mean square error for forecast in generalized integrated autoregressive moving average bilinear model. Though the two models are adequate for forecast when compared with the real series but forecast with one-dimensional integrated autoregressive moving average bilinear model is more adequate.

Key words: Optimal Forecast, Non-linear time Series Models, Bilinear models, Estimation Technique, Mean Square Error.

Introduction

The bilinear time series models have attracted considerable attention during the last years. Overviews of bilinear time series models and their application to various areas and disciplines have been studied [1-9]. The bilinear time series models studied by these researchers could not achieve stationarity for all non-linear series. The one-dimensional integrated autoregressive moving average bilinear time series model that could achieve stationarity for all non-linear time series [11, 12]; and the generalized integrated autoregressive moving average bilinear time series model that could achieve stationarity for all non-linear time series [13, 14] were reported.

Forecasting connote an attempt to see into the future. There are two words, which are used to denote numerical forecasting methods namely forecasting and prediction.

Forecasting is the process of estimation in unknown situations. Prediction is a similar, but more general term, and usually refers to estimation of time series, cross-sectional or longitudinal data. Forecasting is commonly used in discussion of time-series data. Therefore forecasting is a powerful useful instrument in planning and making a wise decision about the future [10]. As a result of feature of stationarity for all non-linear series in one dimensional and generalized integrated autoregressive moving average bilinear time series model we shall attempt to study optimal forecast using these two models and see the one that performs better.

Materials and methods

One-dimensional and generalized integrated autoregressive moving average bilinear time series models

We define one dimensional integrated auto-regressive moving average bilinear time series model as follows:

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$$\phi(B)X_t = \alpha(B)\nabla^d X_t + \beta(B)e_t + \left(\sum_{k=1}^{r} b_{k1} X_{t-k}\right)e_{t-1}, \text{ denoted as BL (p, d, q, r, 1),}$$

where, $\alpha(B) = 1 - \alpha_1 B - \alpha_2 B^2 .... - \alpha_p B^p$, $\beta(B) = 1 - \beta_1 B - \beta_2 B^2 .... - \beta_q B^q$, and

$$X_t = \phi_1 X_{t-1} + ... + \phi_{p+d} X_{t-p-d} + e_t - \beta_1 e_{t-1} - ... - \beta_q e_{t-q} + \left(\sum_{k=1}^{r} b_{k1} X_{t-r}\right)e_{t-1} \quad (1)$$

where, $\alpha_1,...,\alpha_p$ are the parameters of the autoregressive component; $\beta_1,...\beta_q$ are the parameters of the associated error process; $b_{11},.........,b_{r1}$ are the parameters of the non-linear component and d is the degree of consecutive differencing required to achieve stationarity.

We define generalized integrated autoregressive moving average bilinear time series model as follows:

$$\phi(B)X_t = \alpha(B)\nabla^d X_t + \beta(B)e_t + \sum_{k=1}^{r}\sum_{l=1}^{s} b_{kl} X_{t-k} e_{t-l}, \text{ denoted as GBL (p, d, q, r, s)}$$

where $\alpha(B) = 1 - \alpha_1 B - \alpha_2 B^2 .... - \alpha_p B^p$, $\beta(B) = 1 - \beta_1 B - \beta_2 B^2 .... - \beta_q B^q$ and

$$X_t = \phi_1 X_{t-1} + ... + \phi_{p+d} X_{t-p-d} + e_t - \beta_1 e_{t-1} - ...... - \beta_q e_{t-q} + b_{11} X_{t-1} e_{t-1} + ... + b_{rs} X_{t-r} e_{t-s} \quad (2)$$

$\alpha_1,...,\alpha_p$ are the parameters of the autoregressive component; $\beta_1,...\beta_q$ are the parameters of the associated error process; $b_{11},.........,b_{rs}$ are the parameters of the non-linear component and d is the degree of consecutive differencing required to achieve stationary.

Model Estimation

The estimation of the models are similar, we shall report the estimation of generalized type since $m_i = 1,2,3....,s$ for the generalized case include $m_i = 1$ the one dimensional case. Suppose that X_t are generated by equation (1), the sequence of random deviates $\{e_t\}$ could be determined from the relation

$$e_t = X_t - \phi_1 X_{t-1} - ...... - \phi_{p+d} X_{t-p-d} + \beta_1 e_{t-1} + ... + \beta_q e_{t-q} - b_{11} X_{t-1} e_{t-1} - .... - b_{rs} X_{t-r} e_{t-s} \quad (3)$$

To estimate the unknown parameters in equation (3), we make the following assumptions:

(i) The errors $\{e_t\}$ are independent and identically distributed with mean zero and variance σ^2 with finite kurtosis.

(ii) The values of $|\phi 's| < 1$ and $|b_{kl}'s| < 1$ ensure that stationarity and invertibility conditions required of the bilinear process are satisfied. For details see [11].

Thus, maximizing the likelihood function is equivalent to minimizing the function $Q(G)$, which is as follows:

$$Q(G) = \sum_{i=m}^{n} e_t^2, \quad (4)$$

with respect to the parameter $G^{'} = (\cdot_{1},....,\cdot_{p};\bar{\ }_{1},\bar{\ }_{2},....,\bar{\ }_{q};B_{11},....,B_{rs})$

Then the partial derivatives of Q(G) are given by

$$\frac{dQ(G)}{dG_i} = 2\sum_{t=m}^{n} e_t \frac{de_t}{dG_i} \qquad \text{(i = 1, 2,…..,R)} \tag{5}$$

$$\frac{d^2Q(G)}{dG_i dG_j} = 2(\sum_{t=m}^{n} e_t \frac{de_t}{dG_i}\frac{de_t}{dG_j} + \sum_{t=m}^{n} e_t \frac{d^2 e_t}{dG_i dG_j})$$

where these partial derivatives of e(t) satisfy the recursive equations

$$\frac{de_t}{d\cdot_i} + \sum_{j=1}^{s} W_t(t)\frac{de_{t-j}}{d\cdot_i} = X_{t-i}, \text{if i = 1, 2, …, p} \tag{6}$$

$$\frac{de_t}{d\bar{\ }_i} + \sum_{j=1}^{s} W_t(t)\frac{de_{t-j}}{d\bar{\ }_i} = e_{t-i,} \qquad \text{if i = 1, 2,….., q} \tag{7}$$

$$\frac{de_t}{dB_{kmi}} + \sum_{j=1}^{s} W_j(t)\frac{de_{t-j}}{dB_{kmi}} = -X_{t-k}e_{t-m} \qquad \text{(k=1,2,…,r ; m}_i\text{ =1,2,…,s)} \tag{8}$$

$$\frac{d^2 e_t}{d\cdot_i d\cdot_i^{'}} + \sum_{j=1}^{s} W_j(t)\frac{d^2 e_{t-j}}{d\cdot_i d\cdot_i^{'}} = 0 \text{ (i, i' = 0, 1, 2, …, p)} \tag{9}$$

$$\frac{d^2 e_t}{d\bar{\ }_i d\bar{\ }_i^{'}} + \sum_{j=1}^{s} W_j(t)\frac{d^2 e_{t-j}}{d\bar{\ }_i d\bar{\ }_i^{'}} = 0 \qquad \text{(i, i' = 0, 1, 2, …, q)} \tag{10}$$

$$\frac{d^2 e_t}{d\cdot_i dB_{kmi}} + \sum_{j=1}^{s} W_j(t)\frac{d^2 e_{t-j}}{dB_{kmi} d\text{¤}_i} + X_{t-k}\frac{d^2 e_{t-mi}}{d\cdot_i} = 0$$

$$\text{(i=0,1,2,…,p ; k}_i\text{ =1,2,…,r; m}_i\text{=1,2,…,s)} \tag{11}$$

$$\frac{d^2 e_t}{d\bar{\ }_i dB_{kmi}} + \sum_{j=1}^{s} W_j(t)\frac{d^2 e_{t-j}}{dB_{kmi} d\bar{\ }_i} + X_{t-k}\frac{d^2 e_{t-mi}}{d\bar{\ }_i} = 0$$

$$\text{(i=1,2,…,q ; k}_i\text{ =1,2,…,r; m}_i\text{=1,2,…,s)} \tag{12}$$

$$\frac{d^2 e_t}{d\cdot_i d\bar{\ }_i} + \sum_{j=1}^{s} W_j(t)\frac{d^2 e_{t-j}}{d\cdot_i d\bar{\ }_i} = 0 \tag{13}$$

$$\frac{d^2 e_t}{dB_{kmi} dB_{kmi}^{'}} + \sum_{j=1}^{s} W_j(t)\frac{d^2 e_{t-j}}{dB_{kmi} dB_{kmi}^{'}} + X_{t-k}^{'}\frac{d^2 e_{t-mi}}{dB_{kmi}} = -X_{t-k}\frac{de_{t-m}}{dB_{kmi}^{'}}$$

$$\text{(k, k' =1,2,…,r ; m}_i\text{ m}_i\text{' = 1,2,…,s)} \tag{14}$$

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$W_j(t)=\sum_{j=1}^{s} B_{ij} X_{t-j}$ We assume $e_t = 0$ (t = 1, 2, …, m-1) and also

$$\frac{de_t}{dG_i}=0, \frac{d^2 e_t}{dG_i dG_j}=0, \qquad \text{(i, j = 1, 2, …, R; t = 1, 2, …, m-1)}$$

From e_t=0 (t= 1, 2, …, m-1),

$$\frac{de_t}{dG_i}=0, \frac{d^2 e_t}{dG_i dG_j}=0, \text{ and}$$

$$\frac{de_t}{dB_{kmi}}+\sum_{j=1}^{s} W_j(t)\frac{de_{t-j}}{dB_{kmi}}=-X_{t-k}e_{t-m}$$

(k=1,2,…,r ; m_i =1,2,…,s), it

follows that the second order derivatives with respect to $\cdot_i$ (i = 0, 1, 2, …, p) and $\bar{}_i$ (i = 0, 1, 2, …, q) are zero. For a given set of values $\{\cdot_i\}$, $\{\bar{}_i\}$ and $\{B_{ij}\}$ one can evaluate the first and second order derivatives using the recursive equations 6, 7, 8 and 14.

Let $V(G)=\frac{dQ(G)}{dG_1}, \frac{dQ(G)}{dG_2}, ..., \frac{dQ(G)}{dG_R}$

and let $H(G)=[d^2Q(G)/dG_i dG_j]$ be a matrix of second partial derivatives. Expanding $V(\mathbf{G})$, near $G=\hat{G}$ in a Taylor series, we obtain $[V(G)]_{\hat{G}=G}=0=V(G)+H(G)(\hat{G}-G)$. Rewriting this equation, we have $\hat{G}-G=-H^{-1}(G)V(G)$, thereby obtaining an iterative equation given by $G^{(k+1)}=G^{(k)}-H^{-1}(G^{(k)})V(G^{(k)})$, where $G^{(k)}$ is the set of estimates obtained at the k^{th} stage of iteration. The estimates obtained by the above iterative equations usually converge. The iteration requires good sets of initial values of the parameters. This is done by fitting the best subset of the linear part of the bilinear model.

Forecasting Structure of the Models

Suppose $\{X_t\}$ is a discrete time series and we wished to predict X_{t_0+h} given the semi-infinite realization $\{X_{s,} s \le t_0\}$. Let the predictor be $\tilde{X}_{t_0}(h)$. Then it is well known that $E\left[X_{t_0+h}-\tilde{X}_{t_0}(h)\right]^2$ is minimum if and only if $\tilde{X}_{t_0}(h)=E\left(X_{t_0+h}/X_{s,} s\le 0\right)$. The evaluation of $\tilde{X}_{t_0}(h)$ from the model depends on the unknown parameters.

Typically, we will substitute the estimates of these parameters, and then calculate the predictors. The predictors thus obtained are denoted by $\tilde{X}_{t_0}(h)$, (h =1, 2,…..) and the error by $\hat{e}_{t_0}(h)=X_{t_0+h}-\tilde{X}_{t_0}(h)$, and the mean sum of squares of the errors of the predictors for the period (t_o + h, t_o + h + 1, …, t_o + h + M) is $\pounds^2_{\hat{e}}(h)=\frac{1}{M}\sum_{j=1}^{M}\hat{e}^2_{t_0+j}(h)$ [10]

Results

To present the application of the model and its forecast, we will use a real time series dataset, the Wolfer sunspot. The scientists track solar cycle by counting sunspots – cool planet-sized areas on the Sun where intense magnetic loops poke through the star’s visible surface. We have used annual sunspot numbers for the years 1730-1879, giving 150 observations [12]. We have employed Akaike Information Criterion (AIC) in model selection and the estimated models are given below.

One-dimensional Integrated autoregressive moving Average Bilinear Time series Model

Fitted Model at t=150

$$\hat{X}_t = 0.667154X_{t-1} - 0.299180X_{t-3} - 0.199718X_{t-5} - 0.330045e_{t-1} - 0.404780e_{t-2} - 0.001573 X_{t-1}e_{t-1} - 0.009150X_{t-2}e_{t-1} - 0.001215X_{t-3}e_{t-1} - 0.0110694X_{t-4}e_{t-1} - 0.009126X_{t-5}e_{t-1} + 0.005051X_{t-6}e_{t-1} - 0.026523X_{t-7}e_{t-1} + 0.016699X_{t-8}e_{t-1}$$

Generalized integrated autoregressive moving average bilinear time series model

Fitted Model at t=150

$$\hat{X}_t = 0.667154X_{t-1} - 0.299180X_{t-3} - 0.199718X_{t-5} - 0.330045e_{t-1} - 0.404780e_{t-2} - 0.001675 X_{t-1}e_{t-1} - 0.016864X_{t-1}e_{t-2} + 0.004643X_{t-1}e_{t-3} + 0.013121X_{t-2}e_{t-1} + 0.006690X_{t-2}e_{t-2} - 0.010201X_{t-2}e_{t-3} - 0.024090X_{t-3}e_{t-1} - 0.008779X_{t-3}e_{t-2}$$

Discussion

It was clear form Table 1 that the one-dimensional integrated autoregressive moving average bilinear time series model has smaller residual variance (174.06) and smaller mean square error (13.99) for the forecast when compared with the residual variance (184.65) and mean square error (14.26) of generalized integrated auto-regressive moving average bilinear time series model. Therefore the performance of one-dimensional integrated autoregressive moving average bilinear time series model is better when used for forecasting.

Table 1: Residual Variance and Mean Square Errors for Forecast (Sunspot Data)

Model performance	One-dimensional integrated autoregressive moving average bilinear model	Generalized integrated autoregressive moving average bilinear model
Residual variance	174.06	184.65
Mean square errors	13.99	14.26

Figure 1 shows the time plot of original series. Figure 2 shows the time plot of forecasts of one-dimensional model while Figure 3 shows the time plot of forecast of generalized model. These graphs were plotted separately to see the movement and direction of the original series and the forecast of the two models under study. The direction and movement of original series and forecast of one-dimensional and generalized models using the time plot were shown in Figure 4.

The adequacy of any model lies in its ability to forecast the data appropriately. The summary statistics in table 2 indicated the closeness of the forecast series gotten from the two models under study. We saw that the sum of the series for the forecast in one-dimensional (6791) and generalized model (6820) were closed to the sum of the original series (7117). The mean values of the one-dimensional (45.27) and generalized (45.47) models were also closed to the mean (47.45) of the original series. In other words, the variation in the sum, mean and standard error of one dimensional (2.65) and generalized (2.66) models compared to the original series (2.94) were not enormous. This was so because of the forecasting performance of the two models considered. The time plots in Figure 4 were produced from the summary statistics especially the sum of each of the series which was on yearly basis. Therefore, the relative performance of the two models compared to the original series is made clear in Figure 4.

Table 2: Summary Statistics of Original Series and Forecast Series of One-dimensional and Generalized Integrated Autoregressive moving Average Bilinear Time Series Models

Summary statistics	Original series	One-dimensional bilinear series	Generalized bilinear series
Sum	7117	6791	6820
Mean	47.45	45.27	45.47
Standard error	2.94	2.65	2.66

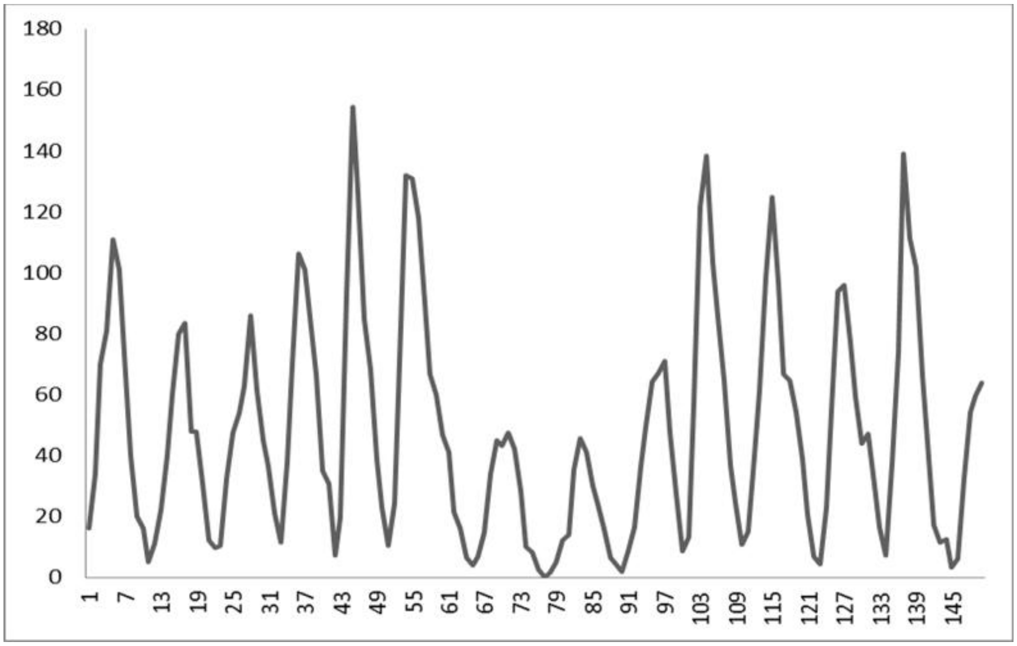

Fig. 1: Time plot of sunspot data (original series).

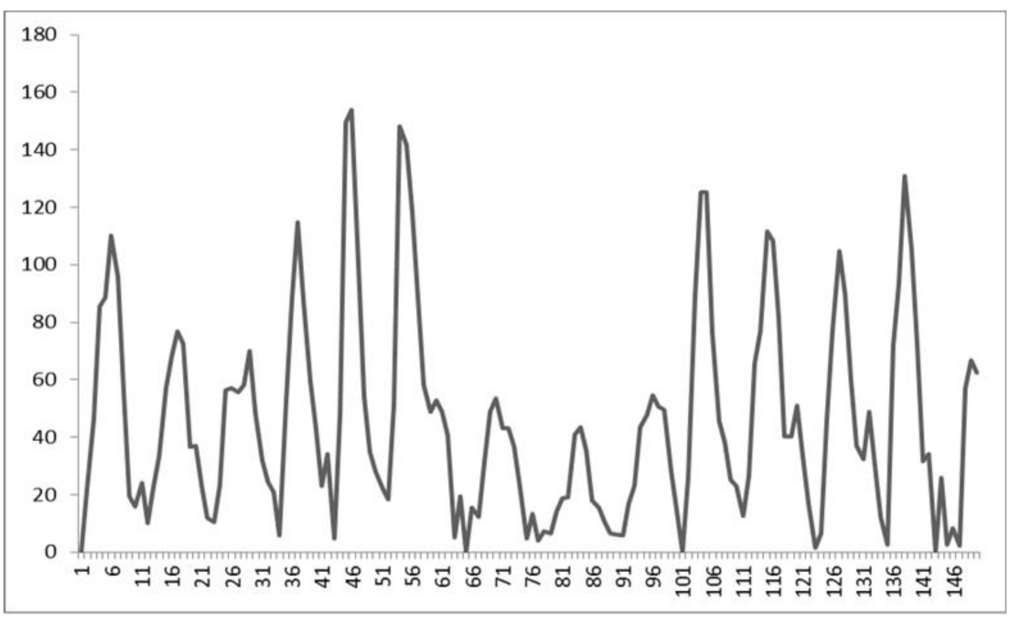

Fig. 2: Time plot of forecast using one-dimensional integrated autoregressive moving average bilinear model.

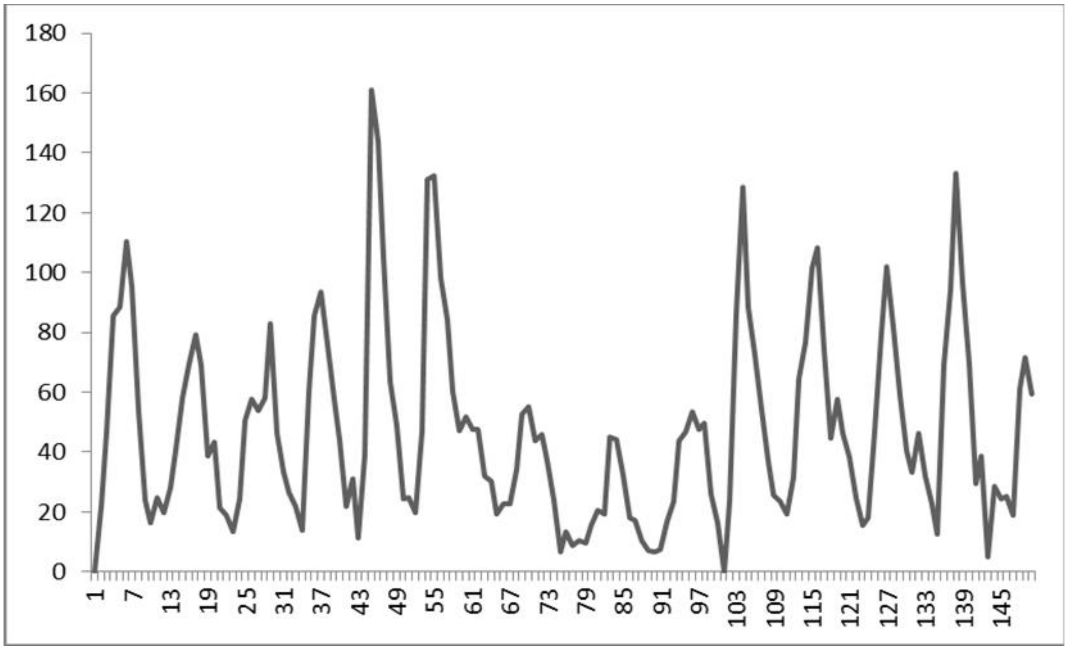

Fig. 3: Time plot of forecast using generalized integrated autoregressive moving average bilinear model.

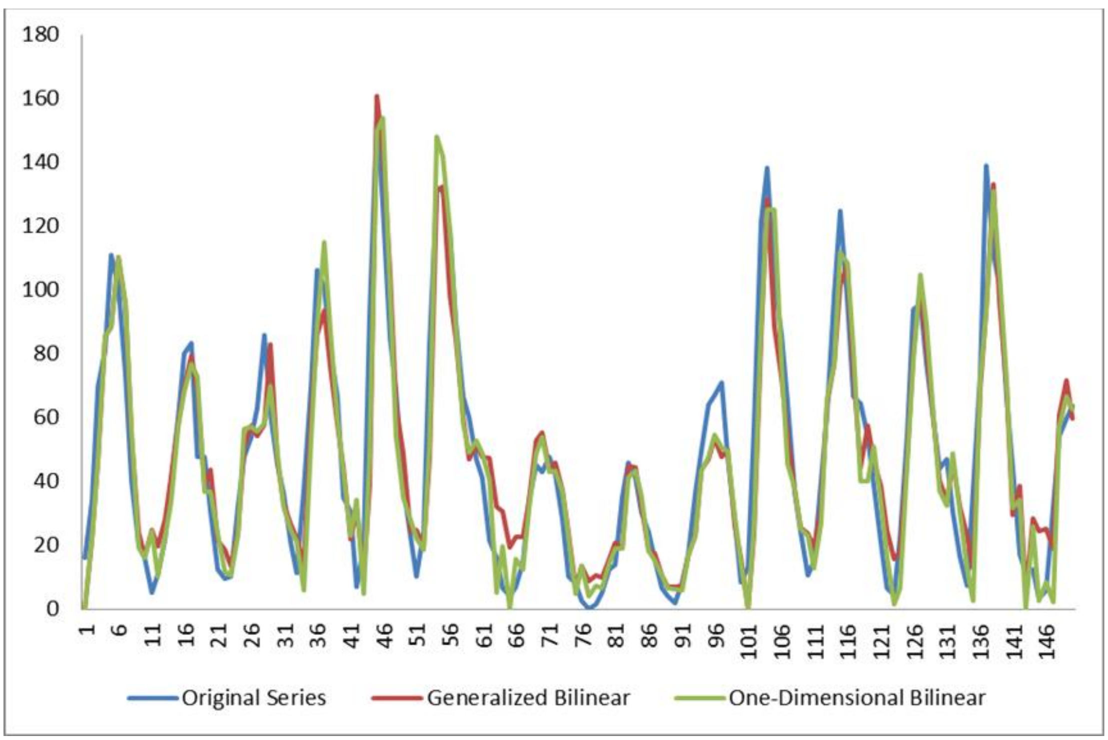

Fig. 4: Time plot of original series, forecast of one-dimensional and generalized integrated autoregressive moving average bilinear models.

Conclusion

Two bilinear time series models that were capable of achieving stationarity for all non-linear series were considered. These two models were used to forecast the future value having estimated their parameters. One dimensional integrated autoregressive moving average bilinear model performed better than generalized integrated auto-regressive moving average bilinear model after we have studied the residual variance attached to the two models. The mean square errors for forecast of the models were studied and we found out that the mean square error attached to one dimensional bilinear model was smaller than generalized model. The two models were used to forecast. On the basis of the forecasting performance, one dimensional integrated autoregressive moving average bilinear time series model has revealed the usefulness of this class of non-linear model for forecasting.

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