

## Investigating structural break-GARCH-based unit root test in US exchange rates

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### Abstract

This paper applied a structural break-GARCH-based unit root test in studying the US exchange rates for twenty-two different countries across America, Europe, Asia-Pacific and Southern Africa. The study employed three different data frequencies – daily, weekly and monthly with a view to understand the dynamics of high frequency series that are characterized by alternating trend patterns and plausible presence of structural breaks. The chosen sample interval included periods of financial crisis or peculiar events. The exchange rates were found to exhibit ARCH effects at higher lags, thus informing the adaptation of the more parsimonious GARCH process in the residuals in contrast to the white noise disturbance assumption. The non-trended and trended structural break-GARCH-based unit root tests' performances were adjudged with other existing tests. With significant break points, from 2 to 5, the presence or otherwise of a unit root in foreign exchange rate series would be better captured when the inherent heteroscedasticity, trend and structural breaks in the series are put into consideration.

**Key words.** Exchange rate, Heteroscedasticity, Unit root, Structural break

### Introduction

The analysis of unit root in time series is very crucial since it helps in characterizing the statistical properties of the series. It is an important part of exploratory data analysis (EDA). As stated in Box *et al.* [1], stationarity (no unit root) has to be ensured in a time series before proceeding to model estimation. Thus, models using Autoregressive Moving Average (ARMA) framework rely heavily on stationarity assumption of time series. Actually, different unit root tests have been proposed; such as those meant for testing non-seasonal unit roots in a series ([2] (ADF test); [3] (PP test); [4] (KPSS test); [5] (Ng-Perron test)); for nonlinear unit root [6] (KSS test)), for seasonal unit root [7,8] (HEGY test) and Structural break unit root test [9] (LP test), [10] (LS test), [5,11] (NP test), [12] (NP, 2010 test)). In all these tests, the assumption of homoscedasticity of the residual term is assumed. Often times, researchers have erroneously applied these tests

wrongly on economic and financial time series data in wrong situations.

The analysis of economic and financial time series may lead to wrong inference once an appropriate and robust unit root test is not applied. The level series (prices/rates) and even the transformed log-returns may display some form of trend and consequently possess structural breaks. In the absence of appropriate unit root analysis in the pre-test, the researcher might obtain unreliable results that leads to wrong inference(s) that could mislead policy makers. For instance, a series known to have a unit root may experience a change in its natural path as a result of some effective government policies, that is, capable of pushing the series away from its long-run trend path [13, 14]. Recently, economic and financial series are being collected and stored at higher frequencies such as daily, weekly and monthly, which often renders the white noise assumption for the ADF type test invalid. Kim



and Schmidt [15] first applied unit root test in the context of heteroscedasticity and observed over-rejection of the unit root test in the presence of Generalized Autoregressive Conditional Heteroscedasticity (GARCH) errors. Other similar unit root tests that are robust to heteroscedasticity are documented in Haldrup [16], Ling and Li [17], Ling *et al.* [18] and Cook [19]. These heteroscedasticity-robust unit root tests are classified as GARCH-based unit root test. These tests allow for the inclusion of GARCH process in the residual of the test regression and thus makes it different from the ADF unit root test with white noise residual. Cook [19] based his findings on the work of Kim and Schmidt [15] and Haldrup [16]. Nevertheless, these earlier versions of GARCH-based unit root tests have their shortcomings in the sense that, they cannot be applied when there are structural breaks in the time series. Applying these tests on high frequency data with inherent structural breaks may render statistical inference(s) invalid [19].

Starting from the proposition of Narayan and Popp [12], the authors developed structural break unit root test by augmenting the classical Dickey-Fuller regression model to account for two endogenous structural breaks of two test specifications: two breaks in the level of a trending data series, and two breaks in the level and slope of a trending data series. By introducing GARCH process to model the residuals of the test regression models, Liu and Narayan (LN) [20] obtained two structural breaks-GARCH-based unit root test that have no intercept and time trend. Narayan and Liu (NL) [21] therefore extended the testing procedure, by including both intercept and time trend components into the modelling framework of Narayan and Liu [20], in order to account for trend as applied in classical ADF-type tests. Narayan *et al.* [22] [NLW thereafter] modified the GARCH-based unit root test to include only the intercept. Following NL [21] and Salisu *et al.* [23], structural break-trend-GARCH based unit root test of NL [21] outperformed other GARCH-based unit root tests at exogenously and endogenously chosen break dates. This test is stable and correctly specified regardless of the way the break date is chosen.

As a contribution to the newly proposed structural break-GARCH-based unit root tests, we subject the tests to more scrutiny on exchange rate data. Exchange rate series are often plagued with serious heteroscedasticity, which often makes the process of statistical inference on the level of stationarity of the series very difficult. Specifically, we apply daily, weekly and monthly US exchange rates to re-validate the unit root tests. Though, Salisu

*et al.* [23] applied the framework on nineteen (19) stock indices in the America, Europe and Asia with a view to ascertain that historical stock indices tend to show significant trend over the years. In our case, we consider using exchange rates since it is often difficult to ascertain the level of stationarity of these economic series.

The rest of the paper is structured as follows: Following the introductory section of this paper, section two focuses on the description of the data with some preliminary analyses. The third section discusses the methodology of the structural break-GARCH-based unit root test, while section four discusses the result of findings of the analyses and performance comparison with existing unit root tests. The final section summarises with some concluding remarks.

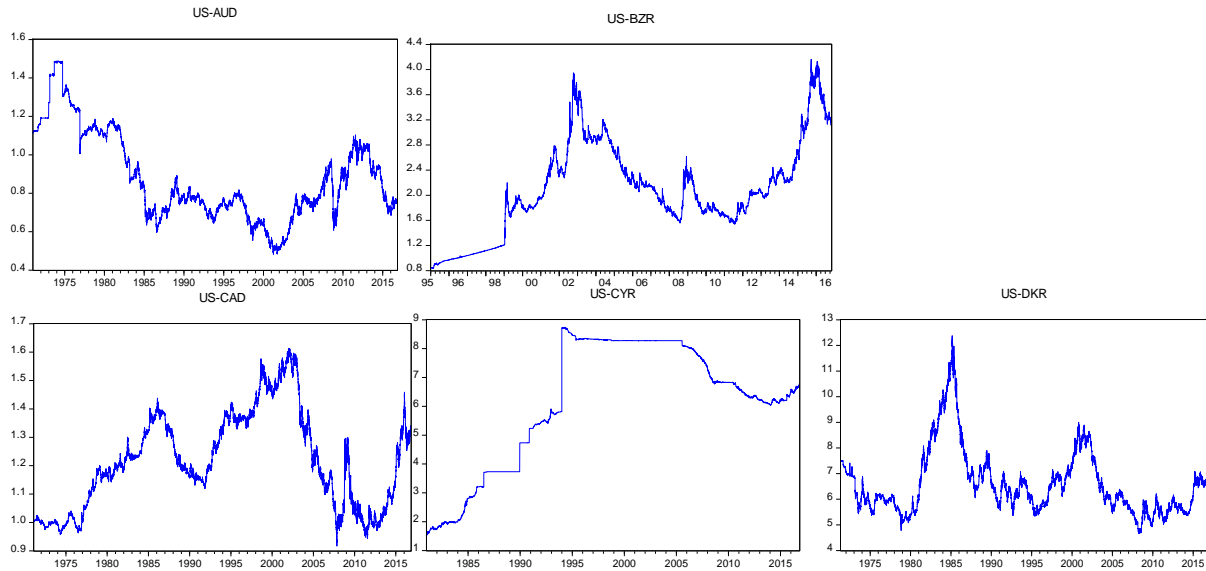
### Data and preliminary analyses

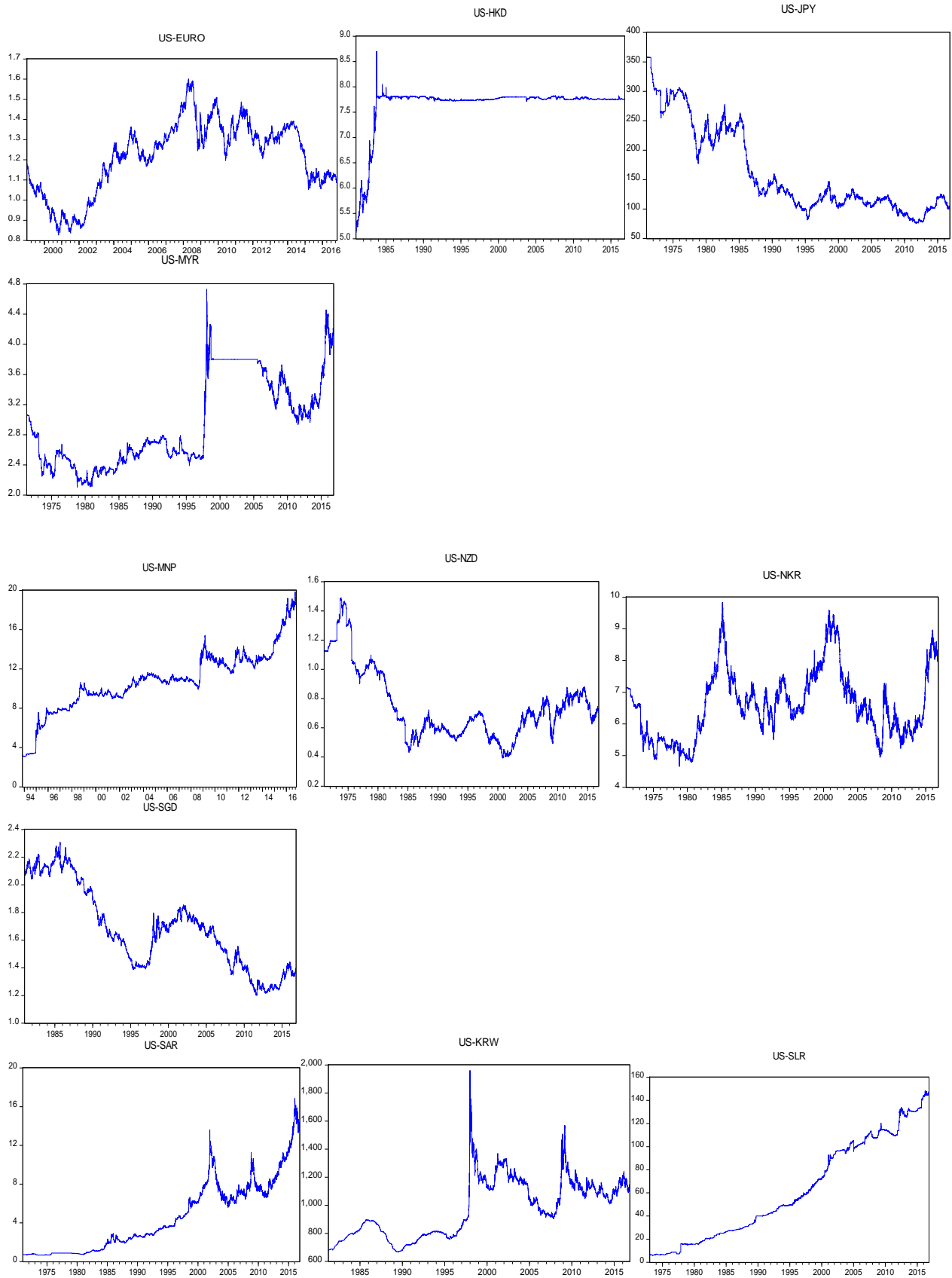
The data considered in this study are the daily, weekly and monthly US exchange rate for 22 countries, cutting across America, Europe, Asia-Pacific and Southern Africa. The data were obtained from Federal Reserve Bank of St Louis Economic Database (<https://fred.stlouisfed.org/>). The time period coverage of 22 exchange rates for the daily, weekly and monthly frequencies are presented in Table 1. The local currencies as well as the foreign exchange initials are presented in columns 2 and 3 of the table. The start and end dates are also presented, with majority of the time series starting around 1971. The periods of each time series were chosen to capture the various financial occurrences/events, such as the Eastern Asian crises (between 1997 and 1998), capital outflows from emerging economies (May to June 2006), US dollars crisis (March 2005), global financial crisis (2008-2009), US terrorist attack (September 2011), oversupply of oil at the international market, which led to the crash in oil price (2015) and the negotiation of UK's exit from the European union (Brexit deal in mid 2016). All these are different plausible sources of structural breaks in the foreign exchange (hereafter, FX) rates. Apart from these factors, FX rates for these 22 countries are determined based on different policies, such as the soft peg arrangement of China (CNY) and Singapore (SGD), pegged arrangement of Hong Kong dollar (HKD) and floating arrangement of Japan (JPY) and South Korea (KRW). Other FX policies adopted by the remaining countries are stabilized and managed arrangements.

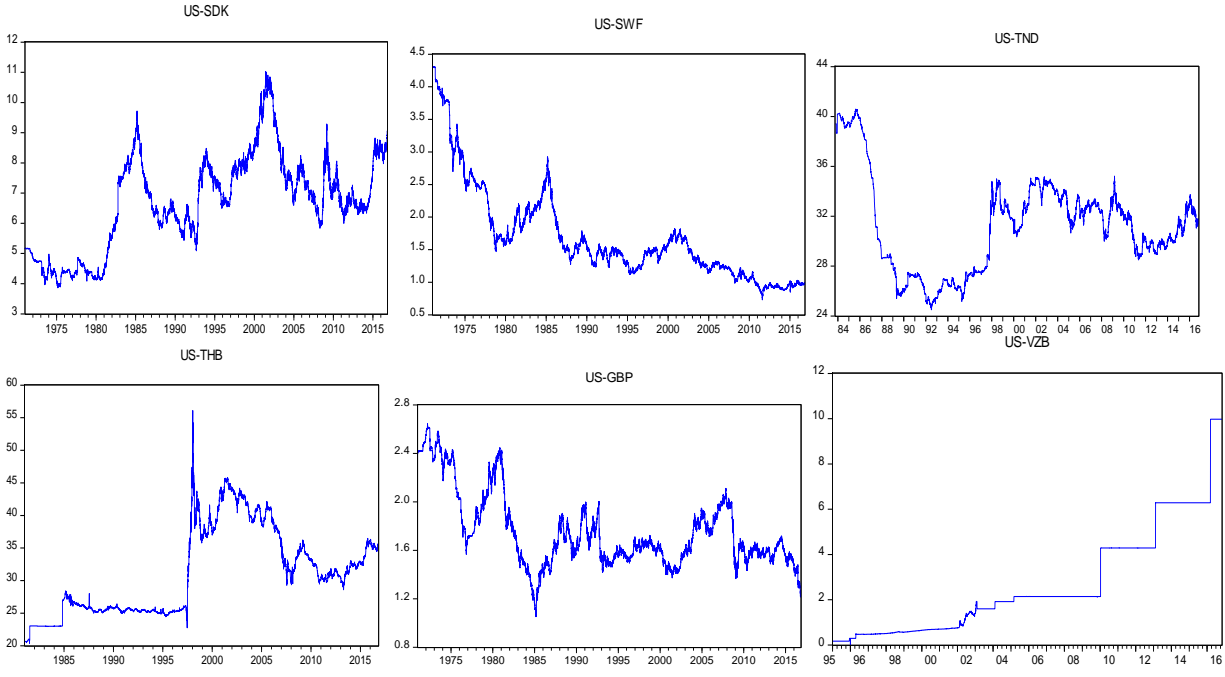
**Table 1.** Data identification and coverage

Country	Currency	FX initial	Daily data		Weekly data		Monthly data	
			Start date	End date	Start date	End date	Start date	End date
Australia	Australian Dollar	AUD	04/01/1971	02/11/2016	08/01/1971	04/11/2016	1971M01	2016M10
Brazil	Brazilian Reals	BZR	02/01/1995	02/11/2016	06/01/1995	04/11/2016	1995M01	2016M10
Canada	Canadian Dollar	CAD	04/02/1971	02/11/2016	08/01/1971	04/11/2016	1971M01	2016M10
	Chinese Yuan	CYR	02/01/1981	02/11/2016	09/01/1981	04/11/2016	1981M01	2016M10
China	Renminbi							
Denmark	Danish Kroner	DKR	30/04/1971	02/11/2016	07/05/1971	04/11/2016	1971M03	2016M10
Europe	Euro	EUR	04/01/1999	02/11/2016	08/01/1999	04/11/2016	1999M01	2016M10
Hong Kong	Hong Kong Dollar	HKD	02/01/1981	02/11/2016	09/01/1981	04/11/2016	1981M01	2016M10
Japan	Japanese Yen	JPY	04/01/1971	02/11/2016	08/01/1971	04/11/2016	1971M01	2016M10
Malaysia	Malaysian Ringgit	MYR	30/04/1971	02/11/2016	07/05/1971	04/11/2016	1971M03	2016M10
Mexico	Mexican New Pesos	MNP	08/11/1993	02/11/2016	07/01/1994	04/11/2016	1993M12	2016M10
New Zealand	New Zealand Dollar	NZD	04/01/1971	02/11/2016	08/01/1971	04/11/2016	1971M01	2016M10
Norway	Norwegian Kroner	NKR	04/01/1971	02/11/2016	08/01/1971	04/11/2016	1971M01	2016M10
Singapore	Singapore Dollar	SGD	02/01/1981	02/11/2016	09/01/1981	04/11/2016	1981M01	2016M10
South Africa	South African Rand	SAR	04/01/1971	02/11/2016	08/01/1971	04/11/2016	1971M01	2016M10
South Korea	South Korean Won	KRW	13/04/1981	02/11/2016	17/04/1981	04/11/2016	1981M05	2016M10
Sri Lanka	Sri Lankan Rupees	SLR	02/01/1973	02/11/2016	05/01/1973	04/11/2016	1973M01	2016M10
Sweden	Swedish Kronor	SDK	04/01/1971	02/11/2016	08/01/1971	04/11/2016	1971M01	2016M10
Switzerland	Swiss Francs	SWF	04/01/1971	02/11/2016	08/01/1971	04/11/2016	1971M01	2016M10
Taiwan	Taiwan New Dollar	TND	03/10/1983	02/11/2016	07/10/1983	04/11/2016	1983M10	2016M10
Thailand	Thai Baht	THB	02/01/1981	02/11/2016	09/01/1981	04/11/2016	1981M01	2016M10
UK	Great British Pound	GBP	04/01/1971	02/11/2016	08/01/1971	04/11/2016	1971M01	2016M10
Venezuela	Venezuelan	VZB	02/01/1995	02/11/2016	06/01/1995	04/11/2016	1995M01	2016M10
	Bolivares							

Source. Federal Reserve Bank of St Louis







**Figure 1.** Plots of daily exchange rates

**Table 2a.** Descriptive statistics for daily exchange rates

Country	FX initial	Mean	Maximum	Minimum	S.D.	Skewness	Kurtosis	JB	ARCH(10)	Trend	Trend1
Australia	AUD	0.8799	1.4885	0.4828	0.2290	0.7043	2.7517	1019.4***	11925.9***	-3.77E-05***	-3.61E-05***
Brazil	BZR	2.0908	4.1638	0.8320	0.7363	0.4246	2.8067	180.1***	5659.6***	0.0002***	-0.0001***
Canada	CAD	1.2162	1.6128	0.9168	0.1683	0.2991	2.1602	528.6***	11888.3***	1.00E-05***	3.98E-05***
China	CYR	6.1347	8.7409	1.5264	2.1886	-0.6645	2.1571	964.8***	9336.1***	0.0005***	0.000482***
Denmark	DKR	6.6144	12.373	4.6605	1.2785	1.5365	5.5452	7876.9***	11820.5***	-9.00E-05***	-0.0003***
Europe	EUR	1.2137	1.6010	0.8270	0.1763	-0.3595	2.3402	184.6***	4616.6***	6.93E-05***	3.64E-05***
Hong Kong	HKD	7.6505	8.7000	5.1270	0.4686	-3.7547	16.119	89005.8***	9326.0***	7.17E-05***	2.44E-05***
Japan	JPY	162.82	358.44	75.720	74.048	0.8999	2.4510	1763.9***	11939.5***	-0.0188***	-0.0055***
Malaysia	MYR	2.9580	4.7300	2.1048	0.5927	0.5023	1.8513	1152.0***	11783.1***	0.0001***	0.0002***
Mexico	MNP	10.791	19.861	3.1022	3.0909	-0.0591	3.8261	174.0***	5975.1***	0.0016***	0.0018***
NewZealand	NZD	0.7468	1.4900	0.3920	0.2383	1.1206	3.7300	2768.0***	11931.1***	-3.67E-05***	-2.51E-05***
Norway	NKR	6.6173	9.8350	4.6585	1.0625	0.5520	2.7827	630.8***	11888.4***	7.72E-05***	7.90E-05***
Singapore	SGD	1.6803	2.3085	1.2007	0.2959	0.3049	1.9737	555.2***	9322.7***	-9.39E-05***	-2.16E-06***
South Africa	SAR	4.5495	16.885	0.6667	3.6294	0.8049	2.8950	1296.6***	11920.6***	0.0010***	0.0008***
South Korea	KRW	975.96	1960.0	667.20	208.02	0.4205	2.4881	374.7***	8834.3***	0.0539***	0.0132***
Sri Lanka	SLR	61.968	148.41	6.0060	42.652	0.3419	1.7125	1012.7***	11424.0***	0.0127***	0.0101***
Sweden	SDK	6.7093	11.037	3.8670	1.5758	0.0693	2.4760	146.3***	11914.5***	0.0003***	-0.0002***
Switzerland	SWF	1.6963	4.3180	0.7296	0.7320	1.5515	5.2263	7267.0***	11942.5***	-0.0002***	-0.0003***
Taiwan	TND	31.227	40.600	24.507	3.8017	0.4976	2.8347	366.1***	8615.8***	-7.76E-05***	-0.0007***
Thailand	THB	31.163	56.100	20.360	6.7707	0.5414	2.1492	738.6***	9267.2***	0.0015***	0.0005***
UK	GBP	1.7533	2.6440	1.0520	0.3095	0.9922	3.3363	2018.2***	11927.5***	-4.97E-05***	-3.94E-05***
Venezuela	VZB	2.6599	9.9750	0.0000	2.3539	1.1854	3.8329	1499.1***	5667.84***	0.0013***	0.0008***

Note, descriptive measurements on the series are presented in the 3<sup>rd</sup> to 9<sup>th</sup> column, and the decision on the normality test is based on the significance of Jarque-Bera (JB) test, where significance of the test implies rejection of null hypothesis of normality. The ARCH Lagrangian Multiplier (LM) test is carried out up to lag 10 in the case of daily and weekly frequency data, while this is carried out up to 5 lags in the case of monthly frequency data. The computed LM chi-squared  $n \cdot R^2$  statistic is reported and significance of ARCH test implies presence of heteroscedasticity in the series. ‘Trend’ presents the coefficient of time trend in an ordinary least squared (OLS) regression of the time series on intercept and time trend. Trend1 is the coefficient of trend term obtained when structural break dummies D1 and D2 for  $\hat{T}_{B1}$  and  $\hat{T}_{B2}$  in Table 4 are included along with time trend in the OLS regression.

\*\*\* indicate significance of all the tests as well as that of trend term at 5% level.

**Source.** Computed by the authors.

**Table 2b.** Descriptive statistics for weekly exchange rates

Country	FX initial	Mean	Maximum	Minimum	S.D.	Skewness	Kurtosis	JB	ARCH (10)	Trend	Trend1
Australia	AUD	0.8798	1.4865	0.4887	0.2291	0.7065	2.7542	204.9***	2355.5***	-0.0002***	-0.0002***
Brazil	BZR	2.0915	4.0971	0.8348	0.7366	0.4226	2.7998	35.8***	1111.9***	0.0012***	-0.0007***
Canada	CAD	1.2158	1.6057	0.9287	0.1683	0.3009	2.1576	106.8***	2360.8***	5.09E-05***	0.0002***
China	CYR	6.1353	8.7318	1.5316	2.1883	-0.6649	2.1580	193.0***	1857.1***	0.0026***	0.0024***
Denmark	DKR	6.6144	12.148	4.6966	1.2785	1.5378	5.5458	1577.4***	2347.4***	-0.0005***	-0.0013***
Europe	EUR	1.2137	1.5880	0.8335	0.1763	-0.3607	2.3380	37.1***	905.5***	0.0003***	0.0002***
Hong Kong	HKD	7.6508	8.5760	5.1360	0.4678	-3.7610	16.148	17878.0***	1853.3***	0.0004***	0.0001***
Japan	JPY	162.64	358.25	75.910	73.858	0.8987	2.4446	352.4***	2363.6***	-0.0937***	-0.0275***
Malaysia	MYR	2.9580	4.4950	2.1128	0.5932	0.5014	1.8442	231.4***	2300.3***	0.0007***	0.0001***
Mexico	MNP	10.844	19.720	3.1051	3.0394	-0.0027	3.8594	36.6***	1173.7***	0.0081***	0.0067***
New Zealand	NZD	0.7464	1.4900	0.3952	0.2382	1.1249	3.7444	559.2***	2347.4***	-0.0002***	-0.0001***
Norway	NKR	6.6174	9.7305	4.7977	1.0625	0.5522	2.7795	126.3***	2338.2***	0.0004***	0.0004***
Singapore	SGD	1.6802	2.2962	1.2037	0.2959	0.3055	1.9735	111.1***	1851.8***	-0.0005***	-1.08E-05
South Africa	SAR	4.5527	16.638	0.6678	3.6311	0.8036	2.8899	258.5***	2356.5***	0.0049***	0.0040***
South Korea	KRW	976.00	1778.6	668.78	207.86	0.4074	2.4159	77.7***	1729.6***	0.2695***	0.0662***
Sri Lanka	SLR	62.084	147.57	6.0278	42.772	0.3347	1.7031	201.9***	2192.2***	0.0635***	0.0601***
Sweden	SDK	6.7105	10.945	3.8978	1.5759	0.0672	2.4750	29.2***	2351.6***	0.0014***	-0.0009***
Switzerland	SWF	1.6952	4.3124	0.7486	0.7305	1.5517	5.2338	1456.6***	2367.6***	-0.0009***	-0.0004***
Taiwan	TND	31.236	40.562	24.556	3.8007	0.4940	2.8362	71.9***	1695.7***	-0.0004***	-0.0035***
Thailand	THB	31.168	53.740	20.493	6.7710	0.5392	2.1424	147.8***	1808.0***	0.0076***	0.0034***
UK	GBP	1.7529	2.6286	1.0651	0.3093	0.9936	3.3436	405.2***	2356.5***	-0.0002***	-0.0002***
Venezuela	VZB	2.6626	9.9750	0.1699	2.3576	1.1854	3.8296	299.6***	1115.1***	0.0065***	0.0034***

Note, descriptive measurements on the series are presented in the 3<sup>rd</sup> to 9<sup>th</sup> column, and the decision on the normality test is based on the significance of Jarque-Bera (JB) test, where significance of the test implies rejection of null hypothesis of normality. The ARCH Lagrangian Multiplier (LM) test is carried out up to lag 10 in the case of daily and weekly frequency data, while this is carried out up to 5 lags in the case of monthly frequency data. The computed LM chi-squared  $n \cdot R^2$  statistic is reported and significance of ARCH test implies presence of heteroscedasticity in the series. 'Trend' presents the coefficient of time trend in an ordinary least squared (OLS) regression of the time series on intercept and time trend. Trend1 is the coefficient of trend term obtained when structural break dummies D1 and D2 for  $\hat{T}_{B1}$  and  $\hat{T}_{B2}$  in Table 4 are included along with time trend in the OLS regression.

\*\*\* indicate significance of all the tests as well as that of trend term at 5% level.

**Source.** Computed by the authors.

**Table 2c.** Descriptive statistics for monthly exchange rates

Country	FX initial	Mean	Maximum	Minimum	S.D.	Skewness	Kurtosis	JB	ARCH(5)	Trend	Trend1
Australia	AUD	0.8800	1.4855	0.5016	0.2291	0.7041	2.7459	46.9***	533.7***	-0.0008***	-0.0007***
Brazil	BZR	2.0906	4.0556	0.8412	0.7360	0.4193	2.7927	8.1***	241.1***	0.0053***	-0.0030***
Canada	CAD	1.2158	1.5997	0.9553	0.1683	0.3015	2.1551	24.6***	527.3***	0.0002***	0.0009***
China	CYR	6.1341	8.7251	1.5518	2.1919	-0.6639	2.1557	44.3***	422.3***	0.0115***	0.0104***
Denmark	DKR	6.6177	11.807	4.7335	1.2762	1.5356	5.5402	362.7***	524.8***	-0.0020***	-0.0059***
Europe	EUR	1.2137	1.5759	0.8525	0.1761	-0.3704	2.3354	8.8***	194.1***	0.0015***	0.0008***
Hong Kong	HKD	7.6499	8.0948	5.1825	0.4696	-3.7659	16.113	4092.3***	415.0***	0.0016***	0.0005***
Japan	JPY	162.85	358.02	76.640	74.101	0.8993	2.4492	81.0***	537.1***	-0.4086***	-0.1203***
Malaysia	MYR	2.9582	4.4093	2.1220	0.5916	0.4989	1.8446	53.2***	478.2***	0.0028***	0.0052***
Mexico	MNP	10.809	19.243	3.1078	3.0662	-0.0468	3.8337	8.0***	262.2***	0.0353***	0.0294***
New Zealand	NZD	0.7469	1.4864	0.3990	0.2385	1.1203	3.7200	126.9***	536.4***	-0.0008***	-0.0006***
Norway	NKR	6.6170	9.4695	4.8167	1.0604	0.5538	2.7803	29.1***	514.5***	0.0017***	0.0017***
Singapore	SGD	1.6805	2.2582	1.2089	0.2960	0.3038	1.9702	25.6***	412.4***	-0.0020***	-4.73E-05
South Africa	SAR	4.5475	16.325	0.6679	3.6299	0.8016	2.8829	59.2***	533.4***	0.0211***	0.0171***
South Korea	KRW	976.39	1707.3	669.25	207.31	0.3797	2.3006	18.9***	365.0***	1.1715***	0.2769***
Sri Lanka	SLR	61.949	146.76	6.0467	42.675	0.3417	1.7115	46.6***	519.4***	0.2762***	0.2325***
Sweden	SDK	6.7091	10.793	3.9166	1.5749	0.0661	2.4730	6.7***	528.5***	0.0063***	-0.0040***
Switzerland	SWF	1.6969	4.3053	0.7800	0.7328	1.5528	5.2281	334.8***	540.1***	-0.0038***	-0.0019***
Taiwan	TND	31.228	40.500	24.769	3.8031	0.4958	2.8337	16.7***	387.3***	-0.0017	-0.0151***
Thailand	THB	31.160	52.982	20.549	6.7728	0.5375	2.1332	34.1***	344.5***	0.0330***	0.0100***
UK	GBP	1.7533	2.6181	1.0931	0.3092	0.9952	3.3356	93.3***	529.3***	-0.0011***	-0.0009***
Venezuela	VZB	2.6573	9.9750	0.1700	2.3498	1.1704	3.7698	66.2***	250.1***	0.0282***	0.0147***

Note, descriptive measurements on the series are presented in the 3<sup>rd</sup> to 9<sup>th</sup> column, and the decision on the normality test is based on the significance of Jarque-Bera (JB) test, where significance of the test implies rejection of null hypothesis of normality. Following Engle (1982), ARCH Lagrangian Multiplier (LM) test is carried out up to lag 10 in the case of daily and weekly frequency data, while this is carried out up to 5 lags in the case of monthly frequency data. The computed LM chi-squared  $nR^2$  statistic is reported and significance of ARCH test implies presence of heteroscedasticity in the series. ‘Trend’ presents the coefficient of time trend in an ordinary least squared (OLS) regression of the time series on intercept and time trend. Trend1 is the coefficient of trend term obtained when structural break dummies D1 and D2 for  $\hat{T}_{B1}$  and  $\hat{T}_{B2}$  in Table 4 are included along with time trend in the OLS regression.

\*\*\* indicate significance of all the tests as well as that of trend term at 5% level.

**Source.** Computed by the authors.



Exchange rate is the ratio of the local currency to a unit of the US dollar. Thus, an increase in the FX rate of a particular country implies a depreciation of the local currency as compared to a unit of US dollar, while a decrease in the FX rate implies an appreciation of the local currency against a unit of US dollar. The dynamics of the time series over the years are given in Figure 1, in which we observe occasional upward and downward patterns in the FX rates. These occasional upward and downward patterns are likened to trending time series.

Descriptive statistics of the series are provided in Tables 2a, 2b & 2c. These statistics, which include: mean, minimum, maximum and standard deviation values of the foreign exchange rates for each country across the different data frequencies, reflect virtually similar estimates. Similar results were also observed for the skewness and kurtosis, implying that the descriptive measurements are not expected to be distinctly different with regard to the choice of data frequencies.

Looking at the skewness and kurtosis statistics across all countries, majority are positively skewed, except for a few cases as seen in the results for China, Europe, Hong Kong and Mexico that were negatively skewed across the three data frequencies. Evidence of leptokurticity was found in the case of Denmark, Hong Kong, Mexico, New Zealand, Switzerland, UK and Venezuela. Furthermore, the Jarque-Bera (JB) test, which is a formal test for the normal distribution of the series, was reported in the tables for all data frequencies and this test indicated rejection of the null hypotheses of normality for the different data frequencies, confirming the non-normality of the FX rates.

As a formal pre-test for heteroscedasticity, the result of the autoregressive conditional heteroscedasticity (ARCH) at lag 10 of the residual was reported for the three data frequencies (see Table 2a-2c). Here, the null hypothesis of homoscedastic residuals was tested against the alternative hypothesis of heteroscedastic residuals. The null hypotheses of no ARCH effect in the model residuals across the data frequencies were rejected, implying that the FX series exhibit conditional heteroscedasticity and thus require modelling with a higher order ARCH - the GARCH process. This strengthens the need to allow for a GARCH process in the test regression for unit root contrary to the white noise error assumption.

The next statistical test carried out evaluates the presence of significant trend term in the FX rates. To execute this, each of the series is regressed on a constant and time trend. The obtained coefficients are reported in Tables 2a, 2b & 2c. The significant coefficients imply that the inclusion of the trend term in the unit root

regression is necessary, otherwise, the trend term in the unit root is redundant. The estimated results revealed that all the coefficients were statistically significant at 5% level for all data frequencies and most of them were positive. Thus, the inclusion of the trend term is necessary in the test regression for the GARCH-based unit root. We further evaluated the behaviour of the trends by accounting for possible structural breaks in the series. In other words, we are trying to verify whether the trend coefficients obtained in the original trend regression are sensitive to structural breaks or otherwise. To achieve this, we employ the Bai and Perron [24], which is an endogenous structural break test, to determine the break points for the foreign exchange rate series. The report of the Bai and Perron [24] test is reported in Table 4. With the exception of the foreign exchange market involving monthly data that has two structural breaks (UK), virtually all the series across all the data frequencies have at least three structural breaks. The results of the extended trend regression are reported as Trend 1 in Tables 2a, 2b & 2c. The results revealed that all trend term coefficients maintained their statistical significance and sign even after the inclusion of structural breaks. Therefore, the behaviour of the trend term is robust to structural breaks.

*The Structural break-GARCH-based Unit root tests*

Following from the preliminary results of the ARCH test and structural breaks, we present the methodologies of the structural break-GARCH-based unit root test. We also present, independently, the methodology of GARCH-based unit root test of Cook [19] and structural break-unit root test of NP [12].

As a follow-up to two-structural break-unit root tests, proposed independently by Lumsdaine and Papell [9] and Lee and Strazicich [10], NP [12] proposed similar two-structural break-unit root test, which differed from other similar tests in the approach with which it selected the break dates. The test regressions for NP [12] are combined as:

$$\Delta X_t = \alpha_0 + \theta_1 DU_{1,t-1} + \theta_2 DU_{2,t-1} + \alpha_1 t + \gamma_1 DT_{1,t-1} + \gamma_2 DT_{2,t-1} + \delta_1 D(T_{B1})_{it} + \delta_2 D(T_{B2})_{it} + \beta X_{t-1} \quad (1)$$

$$+ \sum_{j=1}^k \xi_j \Delta X_{t-j} + \varepsilon_t$$

where  $\gamma_1 = \gamma_2 = 0$ ,  $DU_{i,t} = 1(t > T_{B,i})$  and  $DT_{i,t} = 1(t > T_{B,i}) \cdot (t - T_{B,i})$  are the dummy variables with  $T_{B,i} (i=1,2)$  as the break dates determined by the structural break test. The parameters  $\alpha_0$  and  $\alpha_1$  are the intercept and time trend coefficients, respectively, while  $\beta$  is the coefficient of  $X_{t-1}$  as applied in the classical ADF unit root test [2], with the

augmentation  $\sum_{j=1}^k \xi_j \Delta X_{t-j}$ , where two model

constructs are plausible-model M1 characterized by zero trend coefficients  $\gamma_1 = \gamma_2 = 0$  and model M2-the full model which accounts for breaks in both levels of the time series and slope (trend). Thus, the null hypothesis  $H_0 : \beta = 0$  for unit root is tested against the alternative hypothesis  $H_1 : \beta < 0$  for no unit root.

By excluding the structural break and trend components in the test regression model (1), the model reduces to Cook [19] GARCH-based unit root testing framework,

$$\Delta X_t = \alpha_0 + \beta X_{t-1} + \varepsilon_t \tag{2}$$

where  $\varepsilon_t$  is modelled using the GARCH (1,1) process:

$$\varepsilon_t = z_t \sigma_t^2 \tag{3}$$

$$\sigma_t^2 = \omega + \theta \varepsilon_{t-1}^2 + \phi \sigma_{t-1}^2 \tag{4}$$

where  $\varepsilon_t \approx NID(0,1)$ ;  $\omega > 0$ ;  $\theta \geq 0$  and  $\phi \geq 0$ .

Tables 3a-3c therefore present the results obtained from Cook [19] and NP [12]. Recall that Cook [19] allowed only the GARCH error in the test regression model, while NP [12] allowed model error to follow a normal distribution, as well as structural break in the test regression model. Looking at these attributes in exchange rates based on these unit root tests, we observed more rejections of unit roots when GARCH error was considered in the testing procedure of Cook [19] than in the case of classical unit root tests (ADF and PP). For the three time series frequencies (daily, weekly and monthly) considered, Cook [19] test rejected most unit roots, followed by NP [12] test.

**Table 3a.** Pre-unit root tests for daily exchange rates

Country	FX initial	ADF <sub>I</sub>	ADF <sub>I&amp;T</sub>	PP <sub>I</sub>	PP <sub>I&amp;T</sub>	Cook	NP <sub>M1</sub>	NP <sub>M2</sub>
Australia	AUD	-1.5391[0]	-1.6473[0]	-1.5184[10]	-1.6190[10]	-6.33***	-3.37	-3.41
Brazil	BZR	-1.3339[1]	-1.5249[1]	-1.3531[13]	-1.5498[13]	-3.15***	-2.56	-2.60
Canada	CAD	-1.7221[1]	-1.7001[1]	-1.7061[22]	-1.6832[22]	-0.07	-2.38	-2.41
China	CYR	-2.2079[0]	-1.0772[0]	-2.2105[4]	-1.0744[3]	-0.50	-2.49	-2.80
Denmark	DKR	-1.9095[2]	-1.8796[2]	-2.0029[17]	-1.9806[17]	-1.92	-2.67	-2.86
Europe	EUR	-1.3821[1]	-1.3292[1]	-1.4416[12]	-1.4242[12]	-1.12	-3.88	-3.27
Hong Kong	HKD	-6.4085[33]***	-5.6427[33]***	-6.2706[22]***	-5.5516[22]***	-7.49***	-4.39***	-4.34
Japan	JPY	-2.7919[2]	-2.1182[2]	-2.6611[33]	-2.2680[34]	-3.17***	-3.90	-3.86
Malaysia	MYR	-0.9419[34]	-2.8051[34]	-0.6582[29]	-2.5569[29]	-6.80***	-3.27	-2.95
Mexico	MNP	-0.6097[1]	-2.1562[1]	-0.6767[3]	-2.1838[6]	-1.16	-2.67	-4.05
New Zealand	NZD	-1.7022[1]	-1.3514[1]	-1.6936[7]	-1.3347[7]	-2.05	-2.86	-3.84
Norway	NKR	-1.9671[1]	-2.2337[1]	-2.1514[0]	-2.4090[0]	-2.00	-3.07	-3.19
Singapore	SGD	-1.0987[1]	-1.5326[1]	-1.1743[26]	-1.6753[26]	-0.58	-2.25	-2.78
South Africa	SAR	0.3511[9]	-2.1589[9]	0.3724[3]	-2.0737[6]	3.19***	-2.49	-2.69
South Korea	KRW	-2.6876[37]	-3.5755[37]***	-2.3165[18]	-3.0030[20]	-4.37***	-6.17***	-6.71***
Sri Lanka	SLR	-1.8618[15]	-1.8749[15]	1.8072[12]	-1.9174[11]***	-5.37***	NaN	NaN
Sweden	SDK	-1.4328[1]	-2.0287[1]	-1.4629[9]	-2.0656[9]	-1.63	-2.64	-3.08
Switzerland	SWF	-4.1498[1]***	-3.7744[1]***	-3.9534[31]***	-3.7525[31]***	-3.84***	-3.68	-3.34
Taiwan	TND	-2.1551[1]	-2.0654[1]	-2.1809[27]	-2.1044[27]	-5.15***	-1.71	-4.91***
Thailand	THB	-2.1551[1]	-2.2187[29]	-1.8753[28]	-1.9317[28]	-2.67	-5.78***	-5.97***
UK	GBP	-2.0139[0]	-2.2984[0]	-2.0649[17]	-2.3655[17]	1.35	-2.69	-3.62
Venezuela	VZB	1.0776[0]	-1.2065[0]	1.0984[3]	-1.2065[0]	-0.02	-1.96	-3.38
No. of Rejections		2	3	2	3	10	3	3

Note, ADF<sub>I</sub> and ADF<sub>I&T</sub> present t-statistics for ADF tests of unit root for both intercept only and intercept with trend specifications, and similarly to PP tests (PP<sub>I</sub> and PP<sub>I&T</sub>). Both ADF and PP tests are carried out based on automatic selection of lag lengths using minimum information criteria. The optimal lag lengths are given in squared bracket [ ], and acceptance of null hypothesis of the tests implies the presence of unit root in the time series. For critical values of these unit root tests, see MacKinnon [25]. Recall that Cook [19] is a GARCH-based unit root test, specified without both trend and structural break. The NP (2010) test is a structural break-unit root test, specified in two test regression models, M1 and M2. The t-statistics for the tests are reported accordingly and critical values are only reported at 5% level of significance. Based on the range of the structural breaks obtained in Table 4, corresponding critical value for Cook (2008) test is given as -2.861. Critical values for NP (2010)-M1 and NP (2010)-M2tests are -4.064 and -4.544, respectively.

\*\*\* indicate significance of all the tests as well as that of trend term at 5% level.

**Source:** Computed by the authors.

**Table 3b: Pre-Unit root tests for Weekly Exchange rates**

Country	FX initial	ADF <sub>I</sub>	ADF <sub>I&amp;T</sub>	PP <sub>I</sub>	PP <sub>I&amp;T</sub>	Cook	NP <sub>M1</sub>	NP <sub>M2</sub>
Australia	AUD	-1.5823[1]	-1.6554 [1]	-1.6063[19]	-1.7074 [19]	-2.95***	-2.68	-3.11
Brazil	BZR	-1.3524[1]	-1.5480[1]	-1.4690[10]	-1.6840[10]	-2.66	-2.15	-2.03
Canada	CAD	-1.6429[2]	-1.6179[2]	-1.6827[13]	-1.6630[13]	0.01	-1.62	-1.64
China	CYR	-2.2187[0]	-1.0723[0]	-2.2133[3]	-1.0762[2]	-0.45	-2.46	-2.76
Denmark	DKR	-2.0992[1]	-2.0829[1]	-2.0124[9]	-1.9876[9]	1.70	-2.08	-2.29
Europe	EUR	-1.5417[1]	-1.5134[1]	-1.5417[1]	-1.3999[7]	-0.84	-3.59	-2.71
Hong Kong	HKD	-6.1960[19]***	-5.6759[19]***	-6.9058[20]***	-6.0471[21]***	-7.83***	-4.67***	-2.84
Japan	JPY	-2.3510[1]	-2.0091[1]	-2.3383[15]	-2.0319[15]	1.71	-3.04	-3.06
Malaysia	MYR	-0.8892[6]	-2.7102[6]	-0.7677[20]	-2.6029[20]	-6.91***	-3.50	-4.45
Mexico	MNP	-0.7869[1]	-2.3596[1]	-0.7770[5]	-2.3122[6]	-3.83	-1.55	-1.89
New Zealand	NZD	-1.8317[1]	-1.4249[1]	-1.8382[16]	-1.4769[15]	7.75***	-2.37	-3.32
Norway	NKR	-2.0587[1]	-2.3049[1]	-2.1021[16]	-2.3441[16]	-1.35	-2.30	-2.37
Singapore	SGD	-1.1923[1]	-1.7465[1]	-1.1536[11]	-1.6726[11]	-0.92	-1.58	-2.18
South Africa	SAR	0.4121[1]	-2.0775[1]	0.4506[13]	-2.0550[13]	0.87	-1.57	-1.77
South Korea	KRW	-2.4556[3]	-3.2097[3]	-2.3506[17]	-3.0331[18]	-7.17***	-4.43***	-5.06***
Sri Lanka	SLR	1.6119[6]	-1.9773[6]	1.7559[5]	-2.0302[5]	-57.63***	NaN	NaN
Sweden	SDK	-1.5345[1]	-2.1389[1]	-1.5468[14]	-2.1615[14]	0.19	-1.84	-2.32
Switzerland	SWF	-3.8067[1]***	-3.6947[1]***	-3.8024[10]	-3.5988[10]***	-17.94***	-2.60	-2.75
Taiwan	TND	-2.2319[3]	-2.1632[3]	-2.2468[17]	-2.1711[17]	-2.30	1.04	-2.25
Thailand	THB	-1.9399[14]	-2.0137[14]	-1.9210[17]	-1.9989[17]	-4.95***	1.35	1.25
UK	GBP	-2.2577[1]	-2.5383[1]	-2.3156[18]	-2.6396[18]	-0.66	-2.21	-3.18
Venezuela	VZB	1.0160[1]	-1.2798[1]	1.2616[2]	-0.9192[1]	-0.29	-2.22	-4.20
No of rejections		2	2	1	2	8	2	1

Note, ADF<sub>I</sub> and ADF<sub>I&T</sub> present t-statistics for ADF tests of unit root for both intercept only and intercept with trend specifications, and similarly to PP tests (PP<sub>I</sub> and PP<sub>I&T</sub>). Both ADF and PP tests are carried out based on automatic selection of lag lengths using minimum information criteria. The optimal lag lengths are given in squared bracket [ ], and acceptance of null hypothesis of the tests implies the presence of unit root in the time series. For critical values of these unit root tests, see MacKinnon [25]. Recall that Cook [19] is a GARCH-based unit root test, specified without both trend and structural break. The NP (2010) test is a structural break-unit root test, specified in two test regression models, M1 and M2. The t-statistics for the tests are reported accordingly and critical values are only reported at 5% level of significance. Based on the range of the structural breaks obtained in Table 4, corresponding critical value for Cook (2008) test is given as -2.861. Critical values for NP (2010)-M1 and NP (2010)-M2 tests are -4.064 and -4.544, respectively.

\*\*\* indicate significance of all the tests as well as that of trend term at 5% level.

**Source:** Computed by the authors.

Table 3c. Pre-unit root tests for monthly exchange rates

Country	FX initial	ADF <sub>I</sub>	ADF <sub>I&amp;T</sub>	PP <sub>I</sub>	PP <sub>I&amp;T</sub>	Cook	NP <sub>M1</sub>	NP <sub>M2</sub>
Australia	AUD	-1.7294[1]	-1.8861[1]	-1.5480[4]	-1.6601[4]	0.33	-2.70	-3.09
Brazil	BZR	-1.7615[1]	-1.9801[1]	-1.6263[7]	-1.8560[7]	-1.64	-2.42	-2.27
Canada	CAD	-1.8222[1]	-1.7972[1]	-1.7370[9]	-1.7082[9]	0.03	-1.79	-1.53
China	CYR	-2.2365[0]	-1.0222[0]	-2.2072[2]	-1.0222[0]	-0.51	-2.12	-2.45
Denmark	DKR	-2.2341[1]	-2.2182[1]	-2.2366[9]	-2.2272[9]	-1.70	-2.17	-2.36
Europe	EUR	-1.6257[1]	-1.4852[1]	-1.4970[5]	-1.4344[7]	-0.95	-3.64	-2.86
Hong Kong	HKD	-6.4900[10]***	-6.2695[10]***	-6.9717[2]***	-6.1125[3]***	-17.68***	-4.71***	-2.32
Japan	JPY	-2.6047[1]	-2.5033[1]	-2.6157[8]	-2.3980[8]	-12.07***	-4.53***	-4.60***
Malaysia	MYR	-0.8700[1]	-2.7268[1]	-0.8149[6]	-2.6476[6]	11.12***	-3.07	-2.65
Mexico	MNP	-0.9482[2]	-2.6926[1]	-0.7979[2]	-2.3311[4]	-2.10	-1.61	-1.94
New Zealand	NZD	-1.8570[1]	-1.6297[1]	-1.7705[7]	-1.4662[6]	1.34	-2.68	-3.40
Norway	NKR	-2.4019[1]	-2.6494[1]	-2.1680[7]	-2.3978[6]	-1.96	-2.38	-2.40
Singapore	SGD	-1.2657[1]	-1.8539[1]	-1.1010[7]	-1.5603[6]	-3.11***	-1.82	-2.37
South Africa	SAR	0.3864[1]	-2.1633[1]	0.5915[6]	-1.9340[7]	7.23***	-1.38	-1.54
South Korea	KRW	-2.1012[2]	-2.6659[2]	-2.2520[1]	-2.8584[1]	-8.71***	-4.75***	-5.32***
Sri Lanka	SLR	1.1242[3]	-2.2939[3]	1.4880[10]	-2.1046[10]	18.74***	-2.49	-3.09
Sweden	SDK	-1.7693[3]	-2.4508[3]	-1.7316[10]	-2.3789[10]	-0.74	-1.95	-2.41
Switzerland	SWF	-3.8470[1]***	-3.8454[1]***	-3.9247[5]***	-3.7477[5]***	-15.64***	-3.02	-3.16
Taiwan	TND	-2.1569[1]	-2.1119[1]	-2.2909[8]	-2.2163[8]	-1.85	-1.12	-4.47
Thailand	THB	-2.0557[1]	-2.2158[1]	-1.9336[6]	-2.0353[6]	-6.86***	-6.13***	-6.21***
UK	GBP	-2.4722[1]	-2.8799[1]	-2.2775[7]	-2.6241[7]	0.52	-2.39	-3.33
Venezuela	VZB	1.5551[2]	-0.6510[2]	1.5473[3]	-0.5829[1]	-0.44	-1.31	-3.32
No. of Rejections		2	2	2	2	9	4	3

Note, ADF<sub>I</sub> and ADF<sub>I&T</sub> present t-statistics for ADF tests of unit root for both intercept only and intercept with trend specifications, and similarly to PP tests (PP<sub>I</sub> and PP<sub>I&T</sub>). Both ADF and PP tests are carried out based on automatic selection of lag lengths using minimum information criteria. The optimal lag lengths are given in squared bracket [ ], and acceptance of null hypothesis of the tests implies the presence of unit root in the time series. For critical values of these unit root tests, see MacKinnon [25]. Recall that Cook [19] is a GARCH-based unit root test, specified without both trend and structural break. The NP (2010) test is a structural break-unit root test, specified in two test regression models, M1 and M2. The t-statistics for the tests are reported accordingly and critical values are only reported at 5% level of significance. Based on the range of the structural breaks obtained in Table 4, corresponding critical value for Cook (2008) test is given as -2.861. Critical values for NP (2010)-M1 and NP (2010)-M2 tests are -4.064 and -4.544, respectively.

\*\*\* indicate significance of all the tests as well as that of trend term at 5% level.

Source: Computed by the authors.

Then, combining simultaneously the structural break and the heteroscedasticity attributes as applied in NL' [21] structural break-GARCH-based unit root framework, with test regression model using two endogenous breaks, an intercept and a time trend, is given as,

$$\Delta X_t = \alpha_0 + \alpha_1 t + \beta X_{t-1} + \sum_{i=1}^k D_i B_{it} + \varepsilon_i; \quad i = 1, \dots, k \quad (5)$$

where  $X_t$  is the time series under investigation,  $t$  is the time trend; the dummy  $B_{it} = 1$  if  $t \geq T_{Bi}$  and  $B_{it} = 0$ , otherwise, and  $D_i$  are the dummy variable coefficients. The parameters  $\alpha_0$  and  $\alpha_1$  are the intercept and time trend coefficients, respectively,  $\rho$  is the autocorrelation coefficient at lag 1 between  $X_t$  and  $X_{t-1}$  as applied in the classical ADF unit root test [2]. In the absence of time trend  $t$  in the regression test model in (5), we obtained the non-trended structural break-GARCH based unit root regression model of NLW [22] presented

$$\Delta X_t = \alpha_0 + \beta X_{t-1} + \sum_{i=1}^k D_i B_{it} + \varepsilon_i; \quad i = 1, \dots, k \quad (6)$$

$k$  is the number of significant structural breaks ( $k \leq 5$ ). Note, each break subsample contained at least the minimum fraction  $\varepsilon = 15\%$  of the total size of the time series.

Due to the fact that we are considering endogenously determined structural breaks as a result of unknown break dates, we estimated  $T_{Bi}$ , and the resulting estimates of break dates were used for the unit root test. Specifically, for the purpose of this study, we apply Bai and Perron [24] (BP hereafter) multiple structural break (SB) test to determine the break dates, since this approach allows us to determine up to five SBs in the time series. The Bai-Perron test follows the sequential approach in determining the break dates  $T_{Bi}$  ( $i = 1, 2, 3, \dots, k$ ). The first structural break  $l = 1$  with the break date  $T_{B1}$  is determined based on the rejection of the null hypothesis for the F-statistic  $\sup F_T(l+1/l)$ , which is obtained as an

equivalent to the maximum absolute t-value of the break dummy coefficient  $D_1$  obtained as:

$$\hat{T}_{B1} = \arg \max_{\hat{T}_{B1}} \left| t_{\hat{D}_1}(T_{B1}) \right| \quad (7)$$

Then, imposing the first break estimate  $\hat{T}_{B1}$  in the Bai-Perron testing model, we estimate the second break date  $\hat{T}_{B2}$  as,

$$\hat{T}_{B2} = \arg \max_{\hat{T}_{B2}} \left| t_{\hat{D}_2}(\hat{T}_{B1}, T_{B2}) \right| \quad (8)$$

Thus, repeating this process and increasing  $l$  sequentially to determine the remaining break dates based on the F-test until the test  $\sup F_T(l+1/l)$  fails to reject the null hypothesis of any other additional SB. Thus, the break dates  $T_{Bi}$  ( $i = 1, 2, 3, \dots, k$ ) are determined. The first-two sequentially determined SB dates,  $T_{B1}$  and  $T_{B2}$  are then incorporated in the relevant GARCH-based unit root test regression.

Now, applying the results of multiple structural breaks (Table 4) in the GARCH-based unit root frameworks of NLW [22] and NL [21] yields the results presented in Table 5. Based on the rejections of unit roots by these unit root tests, we obtained improved and consistent results that are similar to Cook [19]. The unit root rejections were seven (7) for the case of daily frequency (using NL [21]) and eleven (11) for the case of weekly frequency (using both NLW [22] and NL [21]). Considering Hong Kong and Switzerland exchange rates, NL [21] and NLW [22] tests also indicated unit root in classical unit root tests, as obtained in this work.

With the fact that Cook [19] unit root test did not allow for time trend in the regression test framework, a scenario that is contrary to FX rates trend tests that were significant in the presence of structural breaks, the test therefore lacked some motivations. Thus, we rely on the unit root tests, which simultaneously account for trend, structural breaks and heteroscedasticity.

**Table 4. Bai and Perron (2003) multiple structural breaks test**

Country	FX initial	Daily			Weekly			Monthly		
		$\hat{T}_{B1}$	$\hat{T}_{B2}$	NSB	$\hat{T}_{B1}$	$\hat{T}_{B2}$	NSB	$\hat{T}_{B1}$	$\hat{T}_{B2}$	NSB
Australia	AUD	05/08/1982 (27021.7)	22/03/2007 (4189.0)	4	13/08/1982 (5404.6)	30/03/2007 (838.8)	4	1982M08 (1246.2)	2007M04 (193.0)	3
Brazil	BZR	21/01/1999 (4873.0)	26/07/2013 (1758.7)	5	29/01/1999 (973.4)	02/08/2013 (353.3)	5	1999M02 (223.0)	2013M08 (80.5)	4
Canada	CAD	01/08/1978 4609.1)	10/08/2005 (7348.5)	4	04/08/1978 (933.7)	12/08/2005 (1470.2)	4	1978M02 (215.1)	2005M08 (339.4)	3
China	CYR	19/11/1990 (322285.8)	28/02/2008 (2908.3)	4	23/11/1990 (6450.4)	07/03/2008 (581.3)	4	1990M12 (1479.7)	2008M03 (134.3)	3
Denmark	DKR	06/04/1981 (2837.2)	13/11/2003 (2433.3)	4	10/04/1981 (567.7)	21/11/2003 (486.0)	4	1981M04 (127.8)	2003M10 (112.9)	4
Europe	EUR	06/05/2003 (7906.8)	04/03/2014 (1288.9)	4	09/05/2003 (1583.0)	14/03/2014 (264.1)	4	2003M05 (366.8)	2014M03 (59.2)	4
Hong Kong	HKD	20/05/1986 (5464.7)	03/10/1991 (2738.6)	5	23/05/1986 (1093.7)	04/10/1991 (547.9)	5	1986M05 (128.5)	1991M09 (128.5)	5
Japan	JPY	11/02/1986 (71075.7)	17/11/1977 (10921.8)	5	14/02/1986 (14289.7)	02/12/1977 (2147.0)	5	1986M02 (3270.4)	1977M11 (504.6)	4
Malaysia	MYR	22/09/1977 (51375.9)	01/12/2006 (3854.2)	4	26/09/1997 (10347.9)	08/12/2006 (773.3)	4	1997M10 (2331.5)	2006M12 (177.4)	4
Mexico	MNP	27/12/2002 (7247.5)	24/05/2013 (2648.8)	4	30/01/1998 (1039.5)	10/10/2008 (1475.4)	4	1998M02 (249.1)	2008M10 (333.4)	4
New Zealand	NZD	15/05/1981 (31334.3)	20/09/2004 (3774.3)	4	22/05/1981 (6244.2)	24/09/2004 (755.5)	3	1981M1 (1445.5)	2004M10 (175.3)	3
Norway	NKR	01/07/1982 (4686.0)	23/09/2004 (2389.9)	4	02/07/1982 (941.7)	01/10/2004 (476.8)	4	1982M07 (216.3)	2004M10 (110.7)	4
Singapore	SGD	31/07/1990 (21964.9)	28/09/2007 (11978.7)	3	03/08/1990 (4391.7)	05/10/2007 (2399.3)	3	1990M08 (1009.9)	2007M10 (550.8)	3
South Africa	SAR	10/06/1998 (38263.1)	26/07/1985 (3515.6)	4	12/06/1998 (7647.3)	04/01/1985 (699.9)	3	1998M06 (1763.0)	1985M01 (161.0)	3
South Korea	KRW	28/10/1997 (27349.9)	17/03/2004 (2853.6)	5	07/11/1997 (5509.3)	19/03/2004 (581.6)	5	1997M11 (1284.2)	2004M02 (136.1)	3
Sri Lanka	SLR	15/06/1989 (52432.0)	15/06/1989 (52432.0)	4	14/05/1999 (1751.1)	14/05/1999 (10448.6)	4	1989M06 (403.9)	1999M05 (2404.5)	4
Sweden	SDK	14/06/1982 (18798.7)	29/01/1993 (1608.5)	3	18/06/1982 (3760.7)	05/02/1993 (322.0)	3	1982M06 (867.9)	1993M02 (74.8)	3
Switzerland	SWF	17/11/1977 (21863.4)	29/12/1986 (7686.3)	4	25/11/1977 (4352.3)	02/01/1987 (1538.0)	4	1977M11 (1007.4)	1987M01 (357.6)	3
Taiwan	TND	16/09/1988 (3711.2)	17/10/1997 (9848.3)	4	16/09/1988 (744.7)	24/10/1997 (1965.9)	4	1988M09 (174.4)	1997M10 (457.7)	3
Thailand	THB	18/07/1997 (23413.3)	10/11/2006 (19745.7)	4	25/07/1997 (4684.0)	17/11/2006 (3978.3)	3	1997M08 (1078.0)	2006M11 (939.6)	3
UK	GBP	01/07/1981 (16838.6)	22/09/2003 (419.6)	3	10/07/1981 (3363.1)	26/09/2003 (83.6)	3	1981M07 (781.3)	2003M10 (19.5)	2
Venezuela	VZB	12/01/2010 (19329.9)	22/04/2013 (3839.0)	5	15/01/2010 (3853.6)	26/04/2013 (772.0)	5	2010m01 (885.8)	2013M04 (185.4)	3

**Note:** NSB denotes the number of structural breaks that are significant from the entire time series, computed based on Bai-Perron multiple structural breaks test. The computed F-statistic  $\sup F_T(l+1/l)$  are given in parenthesis. The critical values of this test for the five break dates are  $l = 1, 2, 3, 4, 5$  are 8.58, 10.13, 11.14, 11.83, 12.25, and  $\hat{T}_{B1}$  and  $\hat{T}_{B2}$  denote the two longest break sub-samples.

**Table 5. Results of NLW (2016) for Non-trended structural break-GARCH based unit root test and NL(2015) for trended structural break-GARCH based unit root test**

Country	FX initial	Daily series		Weekly series		Monthly series	
		NLW	NL	NLW	NL	NLW	NL
Australia	AUD	-10.86***	-15.92***	-3.30	-4.14***	-0.64	-1.24
Brazil	BZR	-5.72***	-20.71***	-24.96***	-12.02***	-3.87***	-12.28***
Canada	CAD	-0.88	-0.80	-1.90	-2.45	-1.01	-1.66
China	CYR	-0.89	-0.54	0.94	0.61	-1.05	-0.60
Denmark	DKR	-2.34	-2.25	-2.24	-2.00	-2.92	-2.73
Europe	EUR	-3.41	-3.51***	-3.82***	-4.01***	-4.36***	-4.54***
Hong Kong	HKD	10.58***	10.36***	-7.15***	-6.93***	-15.87***	-14.24***
Japan	JPY	-15.82***	-5.79***	-7.96***	-7.20***	-5.23***	-5.17***
Malaysia	MYR	-13.30***	-41.83***	-7.38***	-13.92***	17.54***	-21.11***
Mexico	MNP	-1.42	-1.13	-5.18***	-3.38	-1.36	-1.43
New Zealand	NZD	1.50	-1.12	20.83***	19.63***	-1.53	-1.24
Norway	NKR	-2.07	-2.15	-2.05	-1.81	-3.21	-3.13
Singapore	SGD	-1.20	-1.24	-1.82	-1.90	-2.46	-2.50
South Africa	SAR	0.75	0.26	8.25***	14.44***	-4.62***	-1.26
South Korea	KRW	-3.32	-6.69***	-5.97***	-5.28***	-7.77***	-4.64***
Sri Lanka	SLR	NaN	NaN	NaN	NaN	-0.98	-0.98
Sweden	SDK	1.03	-5.22***	-0.87	0.09	-2.17	-0.95
Switzerland	SWF	-11.69***	-17.76***	-5.96***	-7.27***	-4.24***	-6.34***
Taiwan	TND	-1.65	-2.00	-1.26	-0.55	-1.24	-0.98
Thailand	THB	-3.87***	-3.81***	-37.98***	-29.26***	-29.26***	-12.29***
UK	GBP	0.76	0.99	-1.08	-2.17	-2.18	-2.21
Venezuela	VZB	-0.02	-0.16	-0.29	-0.02	4.87***	-3.68
No. of Rejections		7	10	11	11	10	8

**Note:** For consistency, we only made inference on the test based on 5% significant levels. Thus, critical values for the daily, weekly and monthly frequency data as obtained in NL (2015) are -2.87, -3.61 and -3.89, respectively. The critical value for NLW (2016) GARCH-based unit root test is -3.66. Statistical significance of the test is therefore denoted by \*\*\*.





## Conclusion

In this study, the structural break-GARCH-based unit root test, which simultaneously accounted for the heteroscedasticity, trend and structural breaks, was applied in contrast to the existing unit root tests that either accounted for trend, heteroscedasticity presence or structural breaks, individually in testing for unit root in the foreign exchange rate (FX) series. We have been able to apply this unit root testing framework in judging the stationarity of the US FX rates for twenty-two (22) different currencies, cutting across America, Europe, Asia-Pacific and Southern Africa using three different data frequencies – daily, weekly and monthly were used in the study, with the duration of the data capturing significant periods of financial crisis and/or some other peculiar events. These events caused some level(s) of shifts, which resulted in structural breaks in the trend pattern of the series. A similar feat was observed in the preliminary analysis for the three different frequencies, whereby the FX rates revealed the presence of heteroscedasticity among residuals and implied that all the FX series exhibited ARCH effect at higher lag. Consequently, our findings indicated the appropriateness of adapting a parsimonious GARCH process in the residuals, in contrast to the white noise disturbance assumption. Also, with significant trend estimates for both the OLS regression (Trend) and the regression with the inclusion of dummies for the structural breaks (Trend1), the importance of the inclusion of a trend term in the model for FX rates cannot be overemphasized.

The non-trended and trended structural break-GARCH-based unit root test out-performed Cook [19] unit root test, which has already been shown to outperform the two NP [12] model constructs - models M1 and M2. This confirms the superiority of the structural break-GARCH-based unit root test over the existing unit root tests. Conclusively, for a better and more improved unit root testing framework, the three essential features: heteroscedasticity, trend and structural breaks, inherent in an FX series must be put into consideration, while testing for unit root hypothesis.

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